Methods for detecting spatial clustering of economic activities using micro-geographic data

Dottorando: Diego Giuliani

Coordinatore Dottorato: Prof. Guido Pellegrini
Relatore: Prof. Giuseppe Espa
Contents

Preface ................................................................. i
Acknowledgments .................................................... iii

Scientific context and motivation

The phenomenon of spatial clustering within the economic theory
1. The determinant factors of spatial clustering: a review of the theoretical approaches .......... 2
2. The primary role of tacit knowledge ................................................................................. 9
References .......................................................................................................................... 9

Detecting spatial clustering: a review of the most popular measures
1. The properties of an ideal measure of spatial clustering ................................................. 14
2. The first generation measures: the Gini index and its variants ...................................... 17
3. The second generation measures: the Ellison and Glaeser index ................................. 21
References .......................................................................................................................... 23

The continuous approach to space
1. Spatial point pattern statistics ......................................................................................... 25
2. Spatial point pattern statistics in economics ................................................................. 35
References .......................................................................................................................... 36

Essay 1: “Detecting the existence of space-time clustering of firms”
1. Introduction: The spatial-temporal analysis of clusters of firms .................................... 39
2. The statistical methodological framework .................................................................... 41
3. Analysing the long run spatial dynamics of firms: the case of ICT industries in Rome (Italy) 1920-2005 ................................................................................................................. 47
4. Discussion and analysis of the economic implications .................................................... 55
5. Conclusions and research priorities .............................................................................. 57
References .......................................................................................................................... 59

Essay 2: “Measuring industrial agglomeration with inhomogeneous K-function: the case of ICT firms in Milan (Italy)”
1. Introduction .................................................................................................................... 65
2. Phenomena behind spatial concentration of firms: spatial heterogeneity and spatial dependence .................................................................................................................. 66
3. The statistical methodological framework .................................................................... 67
4. A simulation study ......................................................................................................... 70
5. A case study: the distribution of ICT manufacturing firms in Milan (Italy) ................. 75
6. Conclusions .................................................................................................................... 78
References .......................................................................................................................... 79
Essay 3: “Weighting Ripley’s $K$-function to account for the firm dimension in the analysis of spatial concentration”

1. Introduction
2. Measuring the spatial concentration of firms disregarding size: the basic $K$-function
3. Measuring the spatial concentration of firms considering size: the mark-weighted $K$-function
4. Discussion and conclusions
References
Appendix: Analytical derivation of the theoretical mark-weighted $K$-function
Preface

This PhD thesis consists of three self-contained but related essays on the topic of empirical assessment of spatial clusters of economic activities within a micro-geographic framework. The tendency of economic activities to be concentrated in a specific territory is well recognized, starting at least from the seminal studies by Alfred Marshall (Marshall, 1920). This spatial behaviour is not fortuitous; by concentrating in some areas firms enjoy a number of advantages, which then have implications for local economic growth and regional disparities and, as a consequence, are object of study in the fields of economics, geography and policy making. It has been recognized, however, that a major obstacle to further comprehension of the agglomeration phenomena of firms is the lack of a method to properly measure their spatial concentration. The most traditional measures employed by economists, indeed, are not completely reliable. Their most relevant methodological limit lies in the use of regional aggregates, which are built by referring to arbitrary definitions of the spatial units (such as provinces, regions or municipalities) and hence introduce a statistical bias arising from the chosen notion of space. This methodological problem can be tackled by using a continuous approach to space, where data are collected at the maximum level of spatial disaggregation, i.e. each firm is identified by its geographic coordinates, say \((x, y)\), and spatial concentration is detected by referring to the distribution of distances amongst economic activities.

The main purpose of the dissertation is to contribute to the development of this kind of continuous space-based measures of spatial clustering. The scientific context and motivation are outlined in depth in the first three chapters. Then the first essay introduces the space–time \(K\)-function empirical tool, proposed in spatial statistical literature, into economic literature in order to detect the geographic concentration of industries while controlling for the temporal dynamics that characterize the localization processes of firms. The proposed methodology allows to explore the possibility that the spatial and temporal phenomena, producing the observed pattern of firms at a given moment of time, interact to provide space–time clustering. The presence of significant space–time interaction implies that an observed pattern cannot be explained only by static factors but that we should also consider the dynamic evolution of the spatial concentration phenomenon. Indeed, for example, new firm settlements may display no spatial concentration if we look separately at each moment of time and yet they may present a remarkable agglomeration if we look at the overall resulting spatial distribution after a certain time period. In general, without knowing the temporal evolution of the phenomenon under study it is not possible to identify the mechanism generating its spatial structure. As a matter of fact, different underlying space–time processes can lead to resulting spatial patterns which look the same. The methodology is illustrated with an application to the analysis of the spatial distribution of the ICT industries in Rome (Italy), in the long period 1920–2005.

The problem of disentangling spatial heterogeneity and spatial dependence phenomena when detecting for spatial clusters of firms is the topic of the second essay, “Measuring industrial agglomeration with inhomogeneous \(K\)-function: the case of ICT firms in Milan (Italy)”. Spatial clusters of economic activities can be the result of two distinct broad classes of phenomena: spatial heterogeneity and spatial dependence. The former arises when exogenous factors lead firms to locate in certain specific geographical zones. For instance, firms may group together in certain areas in order to exploit favourable local conditions, such as the presence of useful infrastructures, the proximity to the communication routes or more convenient local taxation systems. The phenomenon of spatial dependence, which is often of direct scientific interest, occurs instead when the presence of an economic activity in a given area attracts other firms to locate nearby. For instance, the presence of firms with a leading role encouraging the settlement of firms producing intermediate goods in the same area or the incidence of knowledge spillovers driving industrial
agglomeration. This essay suggests a parametric approach based on the inhomogeneous $K$-function that allows to assess the endogenous effects of interaction among economic agents, namely spatial dependence, while adjusting for the exogenous effects of the characteristics of the study area, namely spatial heterogeneity. The approach is also illustrated with a case study on the spatial distribution of the ICT manufacturing industry in Milan (Italy).

The third paper is titled “Weighting Ripley’s $K$-function to account for the firm dimension in the analysis of spatial concentration”. In the methodological context of the continuous space-based measures of spatial clustering, firms are identified as dimensionless points distributed in a planar space. In realistic circumstances, however, firms are generally far from being dimensionless and are conversely characterized by different dimension in terms of the number of employees, the product, the capital and so on. This implies that a high level of spatial concentration can occur, for example, because many small firms cluster in space, or few large firms (in the limit just one firm) cluster in space. A proper test for the presence of spatial clusters of firms should thus consider the impact of the firm dimension on industrial agglomeration. For this respect, the third essay develops a methodology based on an extension of the $K$-function considering firm size as a weight attached to each of the points representing the firms’ locations.
Acknowledgments

First of all I would like to gratefully acknowledge my supervisor, Professor Giuseppe Espa of the University of Trento, who gave me a lot of help and support and played a fundamental role in shaping my vocation and understanding of spatial statistics. Then I would like to express my sincere appreciation to Professor Giuseppe Arbia, University “G. D’Annunzio” of Chieti, for his invaluable guidance, his encouragement and his support throughout all the PhD course. They both co-authored Essay 1, helpfully criticized drafts of Essay 2 and 3 and also involved me in other interesting related research projects. This has resulted in some important experiences as a, say, “research apprentice”. My work also benefited greatly from various discussions I had with Professor Peter Diggle at the University of Lancaster. He helped me a lot in learning and understanding stochastic processes for spatial data. In particular, his supervision has been fundamental for the development of the probabilistic model in Essay 3. For this, I thank him greatly. Finally, thanks are also due to Professor Roberto Zelli, who as director of the PhD programme, always answered my numerous questions and requests with kindness and willingness.
Scientific context and motivation
The phenomenon of spatial clustering within the economic theory

With the birth and intensification of the globalization process, along with the rapid development of transport and communication technologies, we would expect the location decision making of firms to be less dependent on geographical space. Indeed, in recent years, spatial distance has been less and less a limiting factor of the circulation of goods, capital, knowledge and other production resources, suggesting that economic activity would spread over space. Paradoxically, however, the tendency of firms to spatially concentrate seems to have been increasing (see, e.g., Storper and Venables, 2003; Enright, 2000). Paradigmatic examples of spatial clusters of firms are the Italian industrial districts, Silicon Valley, Wall Street, the European high-tech cluster in Dublin and the German automobile industry located in Baden-Württemberg (for an exhaustive list of examples see Enright, 2000). In the literature, this apparent incongruity is referred as the location paradox (Porter, 2000a). A useful way to summarize the different explanations given by economists to this paradox is to review the main approaches to the analysis of the potential determinant factors of industrial agglomeration.

1 The determinant factors of spatial clustering: a review of the theoretical approaches

In the literature can be found several different definitions of spatial cluster of economic activities. Each of them is conceptually related to the different advantages arising by geographic proximity that tend to induce economic operators to locate close to other existing activities and hence to form spatial agglomerations. Economic theory, in its history, has identified at least five wide classes of spatial concentration advantages. The first one, which can be traced back very far in time, focuses on transportation and transaction cost advantages associated with geographic proximity. The second class refers to advantages related to the spatial closeness to institutions and organizations. The third kind of advantages are the so-called agglomeration economies and the fourth is related to the occurrence of an innovative milieu. Finally, advantages in geographic proximity can also be the result of the market conditions.

1.1 Transportation and transaction cost approach

The theoretical approach that considers the transportation cost advantages, arising because of geographic proximity, as a determinant factor of spatial clustering refers to the seminal contributions of von Thünen (1826) and to the following reappraisal of Weber (1920) and Lösch (1954). In order to keep low transportation costs, economic agents may locate their activity spatially close to their suppliers or buyers. This spatial concentration advantage may be particularly relevant for industries and markets where the frequency of delivery of goods and services is high or many suppliers are bond by just-in-time agreements with producers (Sadler, 1994).

More modern approaches take into account also other kinds of costs arising from interactions amongst economic agents, such as search and information costs, bargaining costs and enforcement costs. These are the so-called transaction costs (see, e.g., Williamson, 1999). It has been theorized that geographic proximity may reduce transaction costs (Scott, 1988; Capellin, 1988). In particular, in a situation of proximity, face-to-face contacts amongst counterparts are normally more frequent and easier to arrange and, as a result, uncertainty and risk of opportunistic behaviour are lower. Bargaining, policing and enforcement costs may thus be lower in clusters, which in turn may positively effect the single firms in terms of profitability.

More precisely, the transaction cost approach uses the theoretical analysis tools which have been elaborated by some contributions of transaction costs theories to organization economics and, in turn, applies them in a spatial framework. Following this contributions (see Williamson, 1979
among others), any exchange of goods and services is subject to transaction costs which should be minimised by economic agents in the same way as production costs in order to choose the optimal production level and also, according to Capellin (1988), the optimal location. Transaction costs can affect the decision making process about the internal organisation of firms, the opportunity to focus into few products, to diversify into different lines of activity, to concentrate in one single location or to establish subsidiary plants in different geographical areas and the kinds of forms of contractual relationships with other economic operators (Capellin, 1988).

Williamson (1981) re-elaborated the paradigmatic optimizing problem of the firm to include transaction costs. Essentially, for a given organisation form (\( f \)), the problem is related to the choices about output (\( q \)) and design (\( d \)) that allow to maximize the following profit function:

\[
Pr(q, d, f) = p(q, d)q - Cf(q, d, s) - Gf(q, d)
\]  

(1)

where \( Pr \) represents profit, \( p(q, d) \) denotes the demand curve, \( s \) denotes combinatorial economies of scope and \( Cf \) and \( Gf \) represent the production costs and transaction costs of organisation form \( f \) (Williamson, 1981).

The behaviour of firms, which is addressed to maximize the profit function, is assumed to be characterized by bounded rationality and opportunism (Williamson, 1979, 1981). Bounded rationality refers to the computational limits of the economic agent, which is not able to receive, store, retrieve and transmit information without error (Simon, 1982). Bounded rationality becomes relevant since the economic and social environment where agents interact is characterised by uncertainty and complexity. When it is then impossible to outline a complete and precise decision making process because of high uncertainty and complexity, “approximation should substitute exactness, complete contracting is impossible and incomplete contracting and adaptive, sequential decision making is the best that can be achieved” (Capellin, 1988).

The assumption of opportunism allows to consider that economic agents behave taking into account that counterparts may not be trustworthy and completely reliable. A dishonest counterpart, indeed, may achieve a higher gain by relying on threats, diffusion of selective and biased information or false promises. The choice about the governance mode of transactions, denoted by \( d \) in equation (1), represents the decisional variable that allows to economise on bounded rationality while simultaneously safeguarding transactions against the risk of opportunistic behaviour of counterparts (Williamson, 1981). The possible alternative governance structures of economic activities, \( d \), are the market and the vertical integration within a large company and also other forms of cooperation and collaboration among firms such as networks of interdependent activities and joint venture. In a spatial context, the possible alternative governance structures may consist on different kind of spatial cluster such as “large metropolitan areas, small and intermediate urban centers, diffused non urban settlement patterns, hierarchical or polycentric urban systems” (Capellin, 1988) among others.

A transaction can be characterised by three attributes, which give indication for a proper governance structure: the frequency of occurrence of transaction, the uncertainty about the transaction outcome and the presence of transaction-specific investments (Williamson, 1981). According to Capellin (1988), all these three attributes are sensible to spatial distance. Indeed, first of all, in case of frequent transactions, geographic proximity is needed to minimize transportation and communication costs. Secondly, uncertainty and complexity decrease, and hence transaction costs tend to be lower, when counterparts are located in the same cluster (e.g. an urban center or an industrial district) and hence share “a greater reciprocal knowledge, a similar language and it implies similar technology, similar development level, similar culture and similar socio-political institutions” (Capellin, 1988). Thirdly, when buyers and sellers are bound to each others because of specific investments, the interruption of the contractual relationship by one of the counterparts may generate higher transaction costs for all involved agents. Geographic proximity and spatial
agglomeration, which ease reciprocal knowledge and trust, tend to reduce the risk of such interruption (Capellin, 1988). Furthermore, the establishment of trustful relationships amongst economic agents, which located nearby in order to minimize transaction costs, may also encourage the exchange of tacit knowledge (Nonaka and Takeuchi, 1995; Polanyi, 1966), that is a kind of knowledge that can be transferable only through direct face-to-face interaction (Storper and Venables, 2003), which stimulates innovation. As a result, the flow of tacit knowledge and the increased ability to innovate, in turn, reinforce the tendency of firms to cluster.

1.2 Institutional thickness approach

Research on the determinant factors of spatial clustering has also referred to the insights of the institutional economics theories (see Amin and Thrift, 1995; Scott, 2000; Bassett et al., 2002 among others). This approach takes into consideration the advantage to locate close to institutions and organizations which may affect the production process. Local formal institutions and organizations and spontaneously established habits and practices help the operation of the local economy by facilitating the opportunities of exchange and meeting and hence encouraging untraded interdependencies, which are relationships of trust and reciprocity and tacit codes of conduct between firms (Keeble et al., 1999; Capello, 1999; Huggins, 2000). Untraded interdependencies can be conceived as non-rivalrous and non-excludable services which are intrinsically related to the social, economic and geographical characteristics of the clusters. For example, the flow of tacit knowledge is generally assured by long-term exchanges, embedded routines, norms and habits in the clusters (Scott, 2000; Swyngedouw, 2000).

Furthermore, the “institutional thickness” (Amin and Thrift, 1995) of a cluster allows firms to benefit from a web of supporting organisations such as financial institutions, chambers of commerce, trade associations, training organisations, local authorities, and marketing and business support agencies which reinforce a communal sense of identity within the cluster (Bassett et al., 2002).

In more detail, following the institutional economics approach to spatial clustering phenomena, the achievement of local and regional economic development is highly correlated to the extent of institutional thickness within the area (Gibbs et al., 2001). Since institutions are seen as a critical factor of reduction of information and transaction costs, they became a decisive determinant of the efficiency of markets (Harriss et al., 1995). In local and regional innovation systems, formal institutions can also be critical in determining the rate and direction of innovative activities (Lundvall, 1998). Formal institutions, indeed, help in creating links among economic agents in order to set off collaborative activities according to the needs of innovation (Landabaso et al., 1999).

Even local based associations, a spontaneous and less formal kind of institutions (such as business networks, trade associations, labour unions and civil associations), have a primary role in driving cluster competitiveness (Porter, 1998, 2000a). The literature has given particular attention to the importance of business associations (Best, 1990; Humphrey and Schmitz, 1996; Maskell et al., 1998; Meyer-Stamer, 1997). With intensification of the globalization process, the social and economic environment in which economic agents operate has become more complex and uncertain making needed for firms the access to specialist business services. On local basis, business associations can provide such services, thus contributing to collective efficiency (Helmsing, 2001). They then represent an important component of the institutional thickness of clusters and are an important part of the local social capital (Amin and Thrift, 1994).

As a matter of fact, local institutional thickness can be conceived as a combined set of formal and informal institutional elements, including synergy, collective working for a common purpose and shared values. A high number of different institutions, a high level of interaction amongst
institutions in the cluster and mutual awareness of being in a common enterprise are the factors that contribute towards the creation of institutional thickness (Amin and Thrift, 1995). The local networks and partnerships facilitated by institutional thickness represent important mechanisms to enable economic and social integration (Oliver and Jenkins, 2005). Furthermore, participation and involvement in voluntary associations promote communication and diffusion of information and generate and reinforce trust in societal norms which, in turn, foster co-operation and economic development and political stability (Putnam, 1993; Yeung, 2000).

In the global competitive and technological economy, firms need to continuously innovate. Even though lots of technological knowledge is codified and more and more accessible worldwide, its proper adjustment to local conditions needs tacit knowledge (Maskell et al., 1998; Raco, 1999). Not all firms, however, are autonomously able to intercept and translate in practice tacit knowledge, especially the smaller and less experienced ones. In this circumstance, economic operators need to rely on external resources in order to learn. Institutions, and above all associations, help to develop the preconditions necessary for collective learning (Keeble et al., 1999). Therefore, institutional thickness creates the requirements for firms to learn by interaction (Morgan, 1997; Raco, 1999).

In conclusion, the institutional economics approach considers local institutional endowments as an important factor of the local collectivisation of the economic and social practices which enable regions to prosper in competitive environments (Cooke and Morgan, 1994; Sassen, 1991). Therefore, strong local institutional relationships, which provide the basis for localised social and economic networks, function as a step to regional economic success (Amin and Thrift, 1995).

### 1.3 Agglomeration economies approach

The agglomeration economies approach rooted in the seminal work by Marshall (Marshall, 1920) and has been primarily developed in more recent times by the New Economic Geography (see Krugman, 1991 and Fujita et al., 1999 among others). Following this line of thinking, firms which locate in any kind of agglomerated environment, such as metropolitan cites, urban agglomerations and industrial clusters, can be affected by positive or negative payoffs. The positive payoffs are identified as agglomeration economies, which are a kind of external economies of scale contrasting directly with internal economies of scale. While the latters are cost savings accruing to the single firm because of growth in the size of plant, the formers are increasing returns from size or growth of output in industry generally (Marshall, 1920). External economies of scale occur as impacts, side-effects or spillovers which are usually not reflected in the costs or prices of a particular good or service; in other words, they do not arise as a result of the market mechanism (Kuah, 2002). In this context, such external economies are essentially spatial externalities, which may be defined generally as economic side-effects of geographic proximity amongst economic agents (Bergman and Feser, 1999).

Agglomeration economies can be subcategorized in “localization economies” and “urbanization economies” (Lösch, 1954). These defining categories refer to different compositions of economic activity and have implications for industrial location and innovation (Henderson, 1983).

Localization economies, otherwise labelled as Marshall–Arrow–Romer externalities (Glaeser et al., 1992), are increasing returns external to the firm but internal to the industry within a geographic region and are generally thought to sourced primarily through the three Marshallian forces (Marshall, 1920): labour market pooling, the sharing of a great variety of specialised intermediate goods and services, and knowledge spillovers.

Essentially, labour market pooling implies that firms in clusters may have a better access to workers and at lower recruiting and training costs. This is because, on one hand, firms can employ graduates from local educational institutions that provide the training that is demanded on local basis; and, on the other hand, a spatial cluster of related firms of the same industry may create a pool of specialized skills. Such a situation is advantageous for both firms and workers. For the formers, labour market pooling may decrease the risks of lack of skilled labour; for the workers, the risk of
unemployment is lower because, if economic shocks are not correlated amongst firms, redundant workers in one firm may be absorbed by other local firms (Krugman, 1991). The sharing of intermediate inputs may be vital for industries where the production of goods and services needs to use specialized machinery and services such that one single firm does not represent a sufficiently wide market to grant profitability for a specialized supplier. Indeed, the presence of many local firms creates a wide enough market to incentivize the presence of a great variety of specialized suppliers.

Finally, the third Marshallian force is represented by the knowledge spillovers which are defined as informal exchanges of information and ideas that happen on personal level via face-to-face contacts. This informal diffusion of knowledge is generally more efficient when an industry is concentrated in a rather localized area where workers of different firms can meet each others in their social life and talk freely about their job.

Therefore, within the theoretical framework of localization economies, an industrial spatial cluster “may increase innovation directly by providing industry-specific complementary assets and activities that may either lower the cost of supplies to the firm or create greater specialization in both input and output markets. We expect that industries in which complementary assets are important would more likely be concentrated geographically and realize greater innovative productivity” (Feldman, 2000).

In contrast, urbanization economies are increasing returns external to the firm and the industry but internal to the geographical area where the firm is located (generally the city). This kind of agglomeration benefits source through inter-industry, rather than intra-industry, relations amongst economic agents and are generally associated with city size or density (Lösch, 1954; Feldman, 2000). Lucas (1993) argues that urbanization economies can be considered as the only undeniable reason for the existence of cities by making them more productive. Following Jacobs (1969) these increasing returns driven by geographic proximity are realized through the exchange of complementary knowledge across diverse firms and economic agents within geographic regions. This concept is theoretically equivalent to that of cross-product increasing returns, which occur when an activity increases the marginal product of another activity and the effect is directly related to proximity (Feldman, 2000). Therefore, urbanization economies may potentially decrease search costs and also increase the possibility for fortunate events that would present innovative opportunities (Feldman, 2000).

It has been argued by a consistent part of the literature that knowledge spillovers occur more frequently and are more relevant between local diverse firms and economic agents belonging to different industries rather than between firms of a same core industry implying that spatial clustering of economic activities is primarily driven by urbanization economies rather than localization economies (see Jacobs, 1969; Feldman, 2000; Storper and Venables, 2004 among others).

1.4 Innovative milieus approach

This approach is mainly associated with the Groupe de Recherche Europeen sur les Milieux Innovateurs (GREMI), a research group founded in the mid 1980s with the aim of studying and investigating the phenomena of spatial clustering of innovative activities by referring to the central concept of innovative, or creative, milieu. The GREMI group has defined the innovative milieu as “the set, or the complex network of mainly informal social relationships on a limited geographical area, often determining a specific external ‘image’ and a specific internal ‘representation’ and sense of belonging, which enhance the local innovative capability through synergetic and collective learning processes” (Camagni, 1991).

The innovative milieus approach shares many elements of the other previously outlined approaches, especially the institutional elements, but is the only one to have given specific attention to the role of culture and identity. It indeed assumes a good local institutional endowment in terms of
academies, research laboratories, public support institutions, cooperating firms and other factors as a necessary prerequisite for spatial clustering, but focuses on the elements that make these institutions operate and interact in ways that lead to positive local outcomes and hence incentivize firms to locate in clusters (Fromhold-Eisebith, 2004).

In order to shed more light on the concept of innovative milieu, it may be useful to follow the theoretical scheme proposed by Fromhold-Eisebith (2004). According to such scheme, innovative milieus are characterised by three main sets of elements: effective relationships amongst agents within a local regional structure; social interactions promoting learning processes; image and sense of belonging.

The first class of elements refers to the evidence that cooperation activities and exchanges of information amongst the agents which can contribute to economic development are enhanced by the geographic proximity between them, which make possible easy and frequent face-to-face interactions (Fromhold-Eisebith, 2004). Since creativity has a better chance to occur through combining ideas coming from different fields of activity never associated before (Shapero 1977), in order to promote innovation and development, the actors of an innovative milieu need to be agents of various kind of memberships (manufacturing firms, service firms, universities, research institutions, administrative institutions and industrial promotion institutions among others) (Maillat et al., 1993). Therefore, such actors “can combine complementary capabilities and competencies that are necessary to create new technical solutions or implement new programs” (Fromhold-Eisebith, 2004). GREMI has indeed pointed out that innovative milieus have the important ability of bringing and coordinating economic change and reallocating productive assets (Crevoisier, 2001; Ratti et al., 1997). However, even though the relevant network of personal relationships is restricted to the local region, inflows of knowledge from outside are also important to avoid “seclusion” and to promote the local circulation of information (Fromhold-Eisebith, 2004).

The second set of essential elements of an innovative milieu refers to the advantages arising from collective learning processes. This specific type of advantages is facilitated by solid informal, in most cases private, contacts amongst agents within the local milieu, which are then bonded by strong mutual trust (Fromhold-Eisebith, 2004). The resulting face-to-face kind of communication they establish bring to faster flows of private and non-codified information flows, reduced uncertainty and accelerated learning and innovation (Sweeney, 1987). Furthermore, “the effective combination of personal professional and private relationships does not only provide preferential or costfree access to strategically important news or services but also to emotional support that backs up business decisions to innovate (motivation, encouragement, recognition)” (Fromhold-Eisebith, 2004).

Finally, another critical set of elements defining an innovative milieu is represented by regional image and sense of belonging which refer to the agents’ awareness to be part of a coherent body and to also show such unity to the outside world. This allows to harmonise “the agents’ differing professional background and interests and direct them towards common goals” (Fromhold-Eisebith, 2004). The cluster identity is also highly supported by the unifying role of local culture, such as the technical tradition and the values system (Crevoisier and Maillat, 1991).

1.5 Market conditions approach

Geographic proximity and spatial clustering can not only affect production and industrial relations, but also the markets of the goods and services produced by the firms in the cluster. Porter (1990) argued that agglomeration advantages can be related to the characteristics of the market of the local firms. Economic operators in clusters, indeed, may be subject to strong local rivalry, which can be “highly motivating” and, as a result, may have a positive influence on the productivity and innovative character of firms (Porter, 1990). In this case, the agglomeration advantage is that managers and skilled workers within clusters, in contrast to situations where firms are spatially dispersed, may compete more intensely for immaterial gratification, such as recognition, reputation
or professional pride. Geographical proximity allows firms to better monitor the performance of rivals and, because of that, fosters peer and competitive pressures even among economic operators which are not directly competing on product markets (Porter, 1998).

Firms in clusters may also be positively affected by the presence of relatively sophisticated and demanding local customers which put them under pressure to product goods and services with high standards of quality, features and client assistance (Porter, 1990). Producers that face up to this demand-side pressure may end up to be more competitive than the rivals that do not and, as a result, they will be stimulated to innovate and move to more advanced segments or new more distant customers. The advantages arising because of the home demand pressures are rooted in information and incentives that do not occur when spatial distance is too high. Customers in geographic proximity, indeed, allow for high visibility, easier communication and the possibility for commercial collaboration relationships (Porter, 1998).

Furthermore, geographic proximity may allow to create complementarities among suppliers, which in turn can make buying from a cluster more convenient for costumers (Porter, 1990). Visiting buyers can meet many different sellers in just one trip (Porter, 1998). This can be advantageous for both sellers and costumers. For the formers, the good performance of a single firm may also have a positive influence on the sales of the firms located nearby. On the other hand, costumers have various sources and can switch sellers if they need. Additionally, local proximate suppliers of complementary products can profit from cooperation among each others to maintain the cluster reputation and to benefit from joint activities, such as marketing, research, development and training (Porter, 1998).

Even this approach identifies a strong association between spatial clustering and innovation. Following Porter, who can be considered as the originator of this line of thinking, on one hand, firms within a cluster can perceive more clearly and rapidly new customer needs and take advantage from the agglomeration of firms with buyer knowledge and relationships (Porter, 2000b). Indeed, since knowledge can be generated and transmitted more efficiently in a situation of geographic proximity, firms which production is based on new knowledge may be highly oriented to locate within a cluster (Audretsch, 1998). On the other hand, being within a cluster allows a good position in perceiving new technological, operating or delivery possibilities. Firms in clusters can “learn early and consistently about evolving technology, component and machinery availability, service and marketing concepts, and so on, facilitated by ongoing relationships with other cluster entities, the ease of site visits, and frequent face-to-face contacts. The isolated firm, in contrast, faces higher costs and steeper impediments to acquiring information and a corresponding increase in the time and resources devoted to generating such knowledge internally” (Porter, 2000b).

The ways clusters foster innovation are not only related to the needs and opportunities for innovation that they can potentially create, but refer also to the possibility that spatially aggregated firms can have to act rapidly to turn these opportunities into actual advantages in their operative and planning activities (Porter, 2000b). Indeed, “a firm within a cluster often can more rapidly source the new components, services, machinery, and other elements needed to implement innovations, whether a new product line, a new process, or a new logistical model. Local suppliers and partners can and do get closely involved in the innovation process, thus ensuring that the inputs they supply better meet the firm's requirements. New, specialized personnel can often be recruited locally to fill gaps required to pursue new approaches. The complementarities involved in innovating are more easily achieved among nearby participants” (Porter, 2000b).

The propensity to innovate within a geographically concentrated cluster is essentially reinforced by the sheer, competitive and peer pressure and continuous comparison assured by small spatial distances amongst local economic agents (Porter, 2000b). In a narrow geographical context, where strong similarity exists in terms of fundamentals (such as the costs for labour, utility and access to infrastructures) and the presence of competitors is high, the only option left for firms to distinguish themselves is creativity (Porter, 2000b). As a consequence, the importance to innovate is very high.
However, the positive association between spatial clustering and innovation does not occur naturally and straightforwardly; as a matter of fact, in some cases, to be part of a cluster may curb innovation. Indeed, “when a cluster shares a uniform approach to competing, a sort of groupthink often reinforces old behaviours, suppresses new ideas, and creates rigidities that prevent the adoption of improvements” (Porter, 2000b). Glasmeier (1991) and Ponder and St John (1996) argued that, under particular conditions, the tendency to imitation among competitors within a cluster can lead to homogeneity and inertia and then create “macroculture that suppresses innovation” (Ponder and St John, 1996).

2 The primary role of tacit knowledge

In a way or another, all the five outlined approaches consider the informal flows of knowledge that occur through face-to-face personal interactions as an important determinant factor of spatial clustering. Changes in the international economy towards a knowledge-based economy have indeed gradually shifted the basis of a firm’s competitive advantage from static price competition towards a better ability to create knowledge a little faster than the competitors (Porter, 1990; Maskell and Malmberg, 1995). All theoretical approaches to the study of spatial clustering argue that, to a certain extent, creation of knowledge is more easy in clusters, where many specialized workers, firms, customers and institutions are concentrated into a relatively small limited space and where the transmission of tacit knowledge tends to occur more efficiently by direct human interaction (Glaeser et al., 1992; Henderson et al., 1995; Dumais et al., 2002; Van Oort, 2004; Lambooy and Van Oort, 2005).

Since tacit knowledge is a non-pecuniary externality and is not subject to market transactions amongst economic agents, it is not directly observable and identifiable. Therefore, detecting empirically how tacit knowledge is relatively important in driving spatial agglomeration and hence economic development is quite problematic. This shows that empirical research in the spatial aspects of economy is important and can help in the comprehension of crucial phenomena characterizing the contemporary study of economics.

References


Detecting spatial clustering: a review of the most popular measures

Both geographers and economists have always been interested in measuring inequalities across industries, time and space. In the empirical literature, some indices to capture these inequalities have become standard; however, it has been recognized that the ideal index remains to be developed (Combes et al., 2008). A recent discussion in the literature, primarily developed by Combes and Overman (2004), Duranton and Overman (2005) and the seminal contribution of Arbia (1989), has focused on the properties that the most proper, ideal, index should have. Section 1 presents and discusses these defined properties, which in turn can represent a conceptual benchmark framework to evaluate the existing indices. The following sections introduce the most popular approaches applied in the literature.

The first approach, which refers to the first generation of measures to detect spatial clustering, is represented by adaptations of the Gini index, originally developed to measure inequality across individuals (Gini, 1912, 1921), to the context of the spatial concentration of firms belonging to the same industry.

The second generation of measures, probably initiated by the seminal work of Ellison and Glaeser (1997), takes explicitly into account the space, which is completely neglected in the Gini based indices, and tends to control for the underlying industrial concentration.

1 The properties of an ideal measure of spatial clustering

Any empirical descriptive tool, whatever its level of complexity and refinement, has necessarily to rely on clearly defined assumptions. Therefore, in order to interpret properly a statistical measure, it is important to be acknowledged of the implications of the given assumptions. Furthermore, the comparison between the assumptions and the requirements that an ideal measure should have may reveal how “good” a tool is (Combes et al., 2008).

Since economic theories on spatial clustering often focus on the sector or industry level, the first requirement refers to the issue of comparability across industries (Duranton and Overman, 2005).

Property i. Measures of spatial clustering should be comparable across industries.

In other words, a good measure of spatial concentration should be independent from the number of plants in the industry and from the size of the industry; otherwise, it would be impossible to judge whether the industries are agglomerated or not (Fratesi, 2008). To be more concrete, the analyst has to be able to compare the degree of concentration in an industry with that in an other industry. Generalizing this requirement leads directly to the second property, which refers to the general tendency of economic activities to agglomerate (Duranton and Overman, 2005).

Property ii. Measures of spatial clustering should control for the overall agglomeration of economic activity.

When this property holds, the measure is not affected by the productive concentration within an industry among plants belonging to that industry. This means that, for example, the measure does not indicate that a sector is more spatially concentrated in one particular geographic area than in another one just because this area has a higher population and hence more labour and economic activity.

According to Fratesi (2008), this methodological issue is conceptually related to the theoretical distinction between urbanization economies and localization economies, as outlined in the previous chapter. The overall agglomeration of economic activities, indeed, ought to be the result of
Urbanization economies. In contrast, industrial concentration should be attributed to localization economies, which lead to the third property, as defined by Duranton and Overman (2005) and Combes and Overman (2004).

**Property iii.** Measures of spatial clustering should control for industrial concentration.

When a study measures spatial concentration of economic activities within an industry, in fact, it ought to implicitly aim at detecting the degree of genuine clustering tendency, and not of industrial concentration due to purely idiosyncratic factors affecting the spatial distribution of the plants of the industry under study (Duranton and Overman, 2005). In other words, the industry spatial pattern of economic activities should reveal spatial clustering when it is more concentrated than that of the whole economy; that is, the benchmark value of absence of spatial clustering should be represented by spatial randomness conditional on the general spatial distribution of production. In practical terms, this requirement implies that the measure detects relative, rather than absolute, spatial concentration. In such a setting, spatial clustering is then considered as a phenomenon of extra-concentration of one industry with respect to the concentration of the plants in the whole economy. Although developing relative measures allows to control for industrial concentration, however it does not permit a direct comparison of results across different economies or countries (see Haaland et al., 1999 and Mori et al., 2005).

Measures aiming at detecting concentration of economic activities in space need also to be developed within a methodological framework which considers space in an appropriate way. To begin with, along with comparability across industries, it is important to ensure comparability across spatial scales.

**Property iv.** Measures of spatial clustering should be comparable across spatial scales.

For a concentration measure, considering space in an appropriate way means, first of all, allowing for meaningful comparisons of spatial concentration across different levels of spatial scales (Duranton and Overman, 2005). In more concrete terms, this amounts to saying that an analyst has to be able to judge whether the clustering phenomenon of an industry is more intense, say, at the national than the regional level.

In order to understand if this requirement is satisfied, we need to know how the spatial units are defined. As a consequence, another property relative to space, as defined by Combes and Overman (2004), need to be introduced.

**Property v.** Measures of spatial clustering should be unbiased with respect to arbitrary changes to spatial classification.

This property is relevant for measures which make use of aggregated data and hence have to rely on a discretization scheme of space, which on the other hand would be naturally continuous. Following an example similar to that proposed by Combes et al. (2008), suppose that the twenty Italian regions are replaced by twenty different spatial units defined according to a criterion which does not respect the administrative borders but considers other territorial characteristics. In detecting the spatial clustering tendency of an industry, the measure should have the same value under the both discretization schemes of space.

Subdividing a continuous space in a set of discrete spatial units leads to the problem known in the statistical literature as the modifiable unit problem (Yule e Kendall, 1950), which in this context takes the specific form of the modifiable areal unit problems (MAUP), discussed for instance in Arbia (1989). The MAUP effects on the statistical measures give rise to two different manifestations, namely aggregation and scale (Arbia, 1989). To illustrate these concepts, let us consider the stylized examples reported in Figure 1, borrowed form Arbia (2001).
**Figure 1:** An hypothetical spatial distribution of four plants, represented in four different spatial classifications. Subfigures (b) and (c) illustrate the *aggregation* problem. Subfigures (b) and (d) illustrate the *scale* problem. Each dot represents a plant.

The aggregation problem can be described by comparing Figure 1(a), Figure 1(b) and Figure 1(c). Figure 1(a) shows the presence of a strong spatial cluster tendency towards the centre of the study area. Suppose to detect concentration using spatial aggregates and to define spatial units, within which data are aggregated, by the means of a grid of quadrats as in Figure 1(b). A measure which is not unbiased with respect to arbitrary changes to spatial classification would identify absence of spatial clustering. In contrast, referring to the same spatial resolution, but shifting the origin of the grid in the northwest direction as in Figure 1(c), the same measure would identify the maximum level of spatial clustering (Arbia, 2001).

On the other hand, the comparison with Figure 1(c) highlights the scale problem. Using a finer grid of quadrats, as in Figure 1(d), onto the same dataset, a measure which is not unbiased with respect to arbitrary changes to spatial classification would take a value that is intermediate between the situation depicted in Figure 1(b) and the situation depicted in Figure 1(c) (Arbia, 2001).

Under an economic point of view, not respecting this property is problematic since homogeneous *economic* geographic areas seldom coincide with *administrative* geographic areas (such as provinces, regions or states) within which data are generally aggregated (Combes et al., 2008). As a result, it may happen that economic agents interacting spatially on regular basis (such as workers and their workplaces or firms and their suppliers or customers) are split across different administrative spatial units.
Therefore, “changing the definition of spatial units may result in a significant, but artificial, redistribution of economic activity. In other words, such changes can translate into different measures of concentration even though the degree of “real” agglomeration remains unchanged” (Combes et al., 2008).

The following requirement, as defined by Combes and Overman (2004), is a similar property with respect to the industrial classification.

**Property vi.** Measures of spatial clustering should be unbiased with respect to arbitrary changes to industrial classification.

In defining an industrial classification scheme to evaluate spatial clustering of industries, the activities of firms are distributed into a given number of arbitrary sets, which are then considered as separate units. Therefore, adopting an industrial classification scheme may arbitrarily separate closely related economic activities or, on the other hand, grouped together sectors that, in point of fact, are consistently different. Furthermore, the level of disaggregation can be different across sectors. For instance, disaggregation for manufacturing is typically finer than it is for services (Combes and Overman, 2004).

Finally, a good measure of spatial clustering should grant the possibility to inferentially assess the results. Therefore, first of all, an analyst need a well-established benchmark, or null hypothesis, with which compare the empirical values of the measure.

**Property vii.** Measures of spatial clustering should have a well-established benchmark.

From a strictly statistical point of view, the natural null hypothesis for the phenomenon of localization of economic activity would be spatial randomness, and hence the alternative hypothesis would be spatial clustering on one hand, and spatial dispersion on the other hand. Otherwise, if the aim is to detect the relative spatial concentration of an industry, it would be more appropriate to use the distribution of activities of the whole economy. According to Combes and Overman (2004), however, in order to make any statements about the relevant economic theory, using a benchmark grounded in a specific economic model would lead to more proper measures. This would allow to validate specific theoretical frameworks.

If the benchmark is clear, then adopting the proper inferential framework make possible to assess the statistical significance of the values assumed by the measure. Therefore, one last property is needed to define the ideal measure of spatial clustering, as stated by Duranton and Overman (2005).

**Property viii.** Measures of spatial clustering should give an indication of the significance of the results.

Any statistical measure of spatial clustering, in order to be scientifically valuable, should indicate the probability that the difference between the observed spatial pattern of economic activities and its benchmark is due to systematic localization phenomena and not to pure chance.

2 The first generation measures: the Gini index and its variants

The Gini index (Gini, 1912, 1921) is probably the most popular measure to detect inequality (Combes et al., 2008). Originally it was primarily used to measure inequality across individual incomes (see Sen, 1973 among others). Krugman (1991) applied it for the first time in a spatial context and then it has become very popular as an empirical tool to detect the spatial concentration of a given industry referring to a particular variable such as production, employment, or value-added (see for example Krugman, 1991; Audretsch and Feldman, 1996; Brühlhart and Torstensson,
The Gini index can be described as follows. Notations and terminology are borrowed from Combes et al. (2008). Let indicate the level of the referring variable (e.g., employment) in industry \( s = 1, \ldots, S \) and in region \( r = 1, \ldots, R \) with \( x^s_r \). Essentially, the Gini index identifies how the regional shares of each industry \( s \) are distributed across the regions. Therefore, first of all, we need to compute the regional shares, denoted by \( \lambda^s_r \), as follows:

\[
\lambda^s_r = \frac{x^s_r}{x^s},
\]

where \( x^s = \sum_{r=1}^{R} x^s_r \) represents the total employment in industry \( s \).

We then rank the regions in ascending order, respect to their regional shares \( \lambda^s_r \), to draw the location curve. This curve, which is known as Lorenz curve, is constructed by representing the fractions \( n/R \) of the \( n \) regions with the lowest employment shares in industry \( s \) on the horizontal axis, so that the first \( x \)-coordinate is \( 1/R \), the second is \( 2/R \), and so on. On the vertical axis we represent the cumulative shares of employment in industry \( s \) of this \( n \) regions, \( \lambda^s_{r(n)} \), computed as

\[
\lambda^s_{r(n)} = \sum_{r=1}^{n} \lambda^s_r.
\]

In case of a completely homogeneous distribution of employment in industry \( s \), the regional shares of total employment in industry \( s \) are all equal to \( 1/R \) of total employment in the whole economy. As a result, the Lorenz curve coincides with the 45-degree line. On the other hand, in case of a non homogeneous distribution of employment in industry \( s \), that is when greater shares of employment are concentrated in a small number of regions, the Lorenz curve tends to lie below the 45-degree line. In general, the more the employment in industry \( s \) is concentrated across regions, the more the Lorenz curve departs from the 45-degree line. The Gini index is equal to the area lying between the Lorenz curve and the 45-degree line, and takes values ranging from 0, in case of a completely homogeneous distribution, to 0.5, in case employment is concentrated in just a single region. The values of the Gini index are generally multiply by two to normalize it to a scale from 0 to 1. In formula terms, the Gini index for industry \( s \) is represented, under the normalization \( \lambda^s_{r(0)} = 0 \), as

\[
G^s = 1 - \frac{1}{R} \sum_{n=1}^{R} \frac{1}{R} [\lambda^s_{r(n-1)} + \lambda^s_{r(n)}].
\]

In this version, the index identifies absolute concentration, since the same weight \( 1/R \) is attached to each region, and hence the benchmark is represented by a uniform distribution of employment across regions.

It is however possible to modify the Gini index in order to account for relative concentration of industry \( s \) by referring to the comparison of the regional shares of employment in industry \( s \) with the regional shares of total employment, \( \lambda^s_r \), which are computed as follows:

\[
\lambda^s_r = \frac{x^s_r}{x^s}.
\]
where \( x_r = \sum_{s=1}^{S} x_{rs} \) represents the total employment in region \( r \) and \( x = \sum_{s=1}^{S} x' = \sum_{r=1}^{R} x_r \) is the total employment in all regions of the considered area.

We then rank the regions referring to the ratios equal to the regional share of total employment in industry \( s \) divided by the regional share of total employment in all industries, \( \lambda_r^s / \lambda_r \), to construct the Lorenz curve for relative concentration. In this case, the horizontal axis values are the cumulative regional shares of total employment in all industries, computed as \( \lambda_{1(n)} = \sum_{r=1}^{n} \lambda_r \). A completely homogeneous distribution is then still represented by a Lorenz curve coinciding with the 45-degree line, but now it is characterized by the fact that the regional shares of total employment in industry \( s \) are equal to the regional share of employment in all industries.

The relative version of the Gini index for industry \( s \) is formally represented as follows:

\[
G^s = 1 - \sum_{n=1}^{R} \frac{\lambda_r}{\lambda_{1(n)}} \left[ \frac{\lambda_r^s}{\lambda_{1(n)}} + \frac{\lambda_{1(n-1)}}{\lambda_{1(n)}} \right].
\]

Referring to the properties of an ideal measure outlined in Section 1, the Gini index proves not to be a proper empirical tool to detect spatial clustering of economic activities. It indeed does not allow to control for industrial concentration (Property \( iii \)), that is, it does not allow adequate comparisons among industries with different market structures (given by the number and size of firms) (Combes et al., 2008). As showed by Arbia (1989), as a statistical measure, it is strongly dependent on the chosen definition of space (Properties \( vi \) and \( v \)); for example, in fact, subdividing a particular region into more spatial units may change the rank of regions, and hence modify artificially the value assumed by the index (Combes et al., 2008). Furthermore, the Gini index is a measure with no indication of the statistical significance of departures from the benchmark of absence of concentration (Property \( viii \)).

2.1 The Isard, Herfindahl and Theil indices

Other indices, which essentially refer to the same methodological framework of the Gini index, have become “standard” in the empirical literature on spatial clustering of economic activities. The most popular are the Isard index (e.g. Krugman, 1991; Kim, 1995, Batisse, 2002), the Herfindahl index (e.g. Henderson et al., 1995; Holmes and Stevens, 2002) and the Theil index (e.g. Mills and Zandvakili, 1997; Brülhart and Traeger, 2005). Along with the Gini index, even these three measures do not respect Properties \( iii \), \( vi \), \( v \).

In presenting the indices, we will use the employment as the referring variable for measuring concentration. Again, it will be adopted the notations and terminology used by Combes et al. (2008).

2.1.1 The Isard index

The Isard index, in its relative version, detects concentration within an industry by referring to the absolute departures of the observed regional distribution of employment in an industry from the regional distribution of employment in the whole economy:

\[
I^s = \frac{1}{2} \sum_{r=1}^{R} \left| \lambda_r^s - \lambda_r \right|.
\]
The values assumed by the index range from the inverse of the smallest regional share (indicating that the employment is perfectly homogeneously distributed across the regions) to 1 (indicating that all economic activities of the given industry are located in just one region). This implies that the Isard index values scale is clearly affected by the spatial scale and definition of the spatial units (Combes et al., 2008); therefore, Properties $iii$, $vi$ and $v$ are not respected.

**2.1.2 The Herfindahl index**

The Herfindahl index, in its relative version, is the weighted sum of the square of each regional share of employment in the given industry:

$$H^s = \frac{1}{R} \sum_{r=1}^{R} \lambda_r \left( \frac{\lambda_r}{\lambda} \right)^2.$$

Since the Herfindahl index has a values scale ranging from the inverse of the number of regions, which varies depending on the spatial classification, to 1, it does not respect Properties $iii$, $vi$ and $v$ as well.

**2.1.3 The Theil index**

The Theil index is based on the concept of entropy, which has originally born in physics where it is basically referred as a measure of disorder. The general class of entropy indices can be represented as follows,

$$E^s(\alpha) = \frac{1}{\alpha^s - \alpha} \left[ \sum_{r=1}^{R} \lambda_r \left( \frac{\lambda_r}{\lambda} \right)^\alpha - 1 \right],$$

where $\alpha$ is a parameter driving the entity of weights attached to the observations. In particular, for $\alpha > 1$ the observations in the upper tail of the distribution weight more; on the other hand, if $\alpha < 1$, it is up to the observations in the lower tail to be relatively more important. The common choice is to use $\alpha = 1$. The limit of $E^s$ as $\alpha$ approaches 1 is the Theil index, $T^s$. More precisely, following the l'Hôpital’s rule, we have

$$\lim_{a \to 1} E^s(\alpha) = \sum_{r=1}^{R} \lambda_r \lim_{a \to 1} \left( \frac{\lambda_r}{\lambda} \right)^\alpha - 1 = \sum_{r=1}^{R} \lambda_r \lim_{a \to 1} \left( \frac{\lambda_r}{\lambda} \right)^\alpha \ln \left( \frac{\lambda_r}{\lambda} \right) 2\alpha - 1,$$

and hence we obtain the Theil index

$$E^s(1) \equiv T^s = \sum_{r=1}^{R} \lambda_r \ln \frac{\lambda_r}{\lambda}.$$

An interesting characteristic of Theil index is the separability property. In a context where we are considering more countries, it may allow to decompose the degree of concentration of the countries’ regions into a degree of concentration between countries and a degree of concentration across regions within each country (Combes et al., 2008). Formally, the separability property can be described by the following equation,
where \( T_b^s \) is the degree of concentration between countries (which not takes into account the regional dimension) and \( T_w^s \) is the degree of concentration within each country.

Specifically, the “between component” of concentration, \( T_b^s \), can be computed by applying the Theil index at the country level, that is

\[
T_b^s = \sum_{c} \Lambda_c^s \ln \frac{\Lambda_c^s}{\Lambda_c}
\]

where the \( \Lambda \) terms are the country level counterparts of the \( \lambda \) terms. Indeed, \( \Lambda_c^s \) represents the country \( c \)'s share of total employment in industry \( s \), and \( \Lambda_c \) denotes the country \( c \)'s share of total employment in the whole economy. More precisely,

\[
\Lambda_c^s = \frac{X_c^s}{X^s}, \text{ where } X_c^s = \sum_{r \in c} x_r^s \text{ and } \Lambda_c = \frac{\sum_{c} X_c^s}{x}.
\]

On the other hand, the “within component” of concentration, \( T_w^s \), is the average of the regional levels of concentration within each country which is computed as the mean of the countries’ Theil indices, weighted by each country’s share in the total employment in industry \( s \):

\[
T_w^s = \sum_{c} \frac{X_c^s}{X^s} T_c^s
\]

where \( T_c^s \) is the Theil index computed only respect to the regions belonging to country \( c \); formally is given by

\[
T_c^s = \sum_{r \in c} \frac{X_r^s}{\Lambda_c} \ln \left( \frac{\Lambda_r^s / \Lambda_c^s}{\Lambda_r / \Lambda_c} \right).
\]

Although the Theil index has the interesting separability property, it shares the same methodological limits with the previously outlined measures of spatial clustering, namely it does not control for industrial concentration and is not unbiased respect to the space definition. However, it is possible to have indications of its statistical significance (Property \( \text{viii} \)) by relying on the bootstrap methods proposed by Brülhart and Traeger (2005).

3 The second generation measures: the Ellison and Glaeser index

Ellison and Glaeser (1997) have introduced a measure to detect spatial clustering of economic activities which allows to control for industrial concentration. It indeed consists on an index that does not depend on changes in the market structure, as given by the number and size of plants, of the industry under study (Property \( \text{iii} \)). The motivating idea of this measure is that when an industry is characterized by a limited number of plants, employment may inevitably be confined to a small number of regions and then the spatial distribution of the economic activity would appear to be non
homogeneous. The first generation measures identify such inhomogeneity, which is not expression of the localization phenomena that are of scientific interest in the context of spatial economics theories, as spatial concentration. On the other hand, the Ellison and Glaeser index identifies clearly spatial clustering depurated from this inhomogeneity by comparing the observed regional distribution of employment in a given industry with the one that would result if economic agents chose to locate their plants randomly and independently across the considered regions.

The Ellison and Glaeser index is based on an Isard-type measure of gross spatial concentration, denoted as $G'_{EG}$, which is computed as the squared sum of all the differences between the regional share of employment in the given industry and the regional share of employment in the whole economy, that is

$$G'_{EG} = \sum_{r=1}^{R} (\lambda^r_s - \lambda^r) \lambda^r_s \lambda^r_s.$$

Ellison and Glaeser (1997) compute the expected value of $G'_{EG}$ for a situation where plants are distributed randomly and independently across the considered regions and show that

$$E[G'_{EG}] = \left(1 - \sum_{r=1}^{R} \lambda^2_r \right) H^s,$$

where $\left(1 - \sum_{r} \lambda^2_r \right)$ identifies the economic activity across regions and, in this context, $H^s = \sum_i (z^s_i)^2$ is the plant level Herfindahl index of industry $s$ in which $z^s_i$ is the share of plant $i$ in total employment in industry $s$.

As a result, $G'_{EG} = E[G'_{EG}]$ implies that the observed regional distribution of employment in industry $s$ is equivalent to the expected random distribution. In contrast, if $G'_{EG} > E[G'_{EG}]$ (respectively $G'_{EG} < E[G'_{EG}]$) employment in industry $s$ tends to concentrate (respectively scatter) in space.

Ellison and Glaeser (1997) develop their index, $\gamma'_{EG}$, as the magnitude of the discrepancy between $G'_{EG}$ and the random distribution benchmark $E[G'_{EG}]$, indeed:

$$\gamma'_{EG} = \frac{G'_{EG} - E[G'_{EG}]}{\left(1 - \sum_{r=1}^{R} \lambda^2_r \right) - E[G'_{EG}]} = \frac{G'_{EG} - \left(1 - \sum_{r=1}^{R} \lambda^2_r \right)}{\left(1 - \sum_{r=1}^{R} \lambda^2_r \right) \left(1 - H^s\right)}.$$

It is scaled so that it takes values from –1 to 1, where the higher is $\gamma'_{EG}$, the higher is the degree of spatial concentration of the observed pattern of industry $s$. Values close to zero indicate absence of concentration in the sense that employment is only as concentrated as it would be expected if the plant locations would have been chosen randomly.

Although the Ellison and Glaeser index provides an important improvement in the ability to measure spatial clustering respect to the first generation measures, it is still not unbiased respect to the arbitrariness of the definitions of the spatial units (Properties iv, v). The problem is related to the fact that the data onto which the index is computed, that is the regional aggregates, are treated as aspatial sets. In other words, the index is constructed neglecting the relevant phenomena of spatial
heterogeneity and spatial dependence (Arbia, 1989). The former phenomenon implies that the
spatial units are not the same, i.e. they have different propensities of hosting economic activities;
and spatial dependence implies that close spatial units are more similar than distant spatial units and
such similarity increases as closeness increases (Tobler, 1970).
Various variants of the Ellison and Glaeser index have been appeared in the literature, the most
popular ones are perhaps those proposed by Maurel and Sédillot (1999), Devereux et al. (1999) and,
notably, Mori et al. (2005) whose index is specifically built to be statistically testable (Property
viii). However, they all share the same methodological limits as the Ellison and Glaeser index.

References


Arbia G. (1989) *Spatial data configuration in statistical analysis of regional economic and related
problems*, Kluwer Academic Publisher, Dordrecht.

Arbia G. (2001) Modelling the geography of economic activities on a continuous space, *Papers in

Audretsch D. B., Feldman M. P. (1996) R&D Spillovers and the Geography of Innovation and


Brülhart M. (2001) Evolving Geographical Concentration of European Manufacturing Industries,

Brülhart M., Torstensson J. (1996), *Regional Integration, Scale Economies an Industry Location in

Brülhart M., Traeger R. (2005) An account of geographic concentration patterns in Europe,


European Union*, in: Henderson J. V., Thisse J.-F. (Eds.), Handbook of Urban and Regional
Economics, Elsevier, North Holland, Amsterdam.


The continuous approach to space

As shown in the previous chapter, the traditional measures of spatial clustering are far from being ideal. Their most relevant methodological limit lies in the use of regional aggregates, which are built by referring to arbitrary definitions of the spatial units (such as provinces, regions…) and hence introduce a statistical bias arising from the chosen notion of space. This methodological problem has been overcome in some recent works (Arbia and Espa, 1996; Marcon and Puech, 2003, 2010; Duranton and Overman, 2005, 2008; Arbia et al., 2008) by using a continuous approach to space, where data are collected at the maximum level of spatial disaggregation, i.e. each plant is identified by its geographic coordinates \((x, y)\), and spatial concentration is detected by referring to the distribution of distances amongst economic activities. The original contributions of this thesis follow this line of research. The reference statistical framework here is the so called *spatial point pattern statistics*, that will be presented throughout this Section.

1 Spatial point pattern statistics

Spatial point pattern statistics is a specific branch of spatial statistics devoted to analyze the structure and characteristics of patterns formed by objects that are distributed in one-, two- or, at least in principle, three-dimensional space. At the planar level the data consist of a set of spatial coordinates, say \((x, y)\), describing the locations of the objects. Any such data-set is labelled as a *spatial point pattern* and is graphically represented by a map of points distributed within the area of space under study. Figure 1 illustrates three different examples of spatial point patterns in a square area. In particular, the first shows local aggregations of points, which could be due to some form of clustering mechanism or to territorial variation within the considered area. On the other hand, Figure 1\((b)\) depicts a pattern where points are distributed approximately regularly over the area, suggesting that a mechanism may have favoured inhibition amongst points’ locations and encouraged an even spatial distribution.

**Figure 1:** Paradigmatic examples of spatial point pattern: (a) aggregated pattern, (b) regular pattern, (c) random pattern

![Paradigmatic examples of spatial point pattern: (a) aggregated pattern, (b) regular pattern, (c) random pattern](image)

The pattern in Figure 1\((c)\), instead, does not show any kind of systematic structure and might be considered as a *completely random* pattern. The basic key concept which, indeed, represents the starting point for the analysis of any spatial point pattern is the hypothesis of *complete spatial
randomness (CSR) (Diggle, 2003; Cressie, 1993). This benchmark hypothesis for a spatial point pattern asserts, heuristically, that the points have been generated under the two specific conditions of \(i\) uniformity and \(ii\) independence, that is, respectively:

\(i\) constant propensity to host points within the pattern; i.e. the area of the pattern is homogeneous;

\(ii\) absence of spatial interactions amongst points; i.e. each point’s location is independent from the other points’ locations.

Generally the first step in the inferential analysis of a spatial point pattern consists on detecting whether the observed pattern is consistent with the CSR hypothesis. As in other branches of statistics, inferential analysis is primarily conducted by relying on formal stochastic mechanisms generating data. In this specific context, stochastic processes are labelled as spatial point processes.

1.1 Spatial point processes

A spatial point process is a probabilistic law that drives that generation of a countable set of objects, identified by points coordinates \(x_i = (x_{i1}, x_{i2})\), in the plane (Diggle, 2003). Most of the point processes used in practical applications are built under the assumptions of stationarity and isotropy. Essentially, a process is stationary if all its properties are invariant under translations of the area through the space, and it is isotropic if all its properties are invariant under rotation (Diggle, 2003). Such assumptions, however, are not so narrow as it might first appear. In particular, for example, the stationarity assumption can be relaxed by allowing local intensity of points to vary over space according to a specific stochastic model or to the behaviour of some spatially referenced explanatory variables.

Point process based inferential methods for analyzing observed spatial point patterns involve comparisons between empirical summary measures on the data and theoretical summary measures of an underlying point process. The theoretical summary measures are quantities which identify the properties characterizing a point process. The \textit{first-order} properties of a point process are described by an \textit{intensity function} (Diggle, 2003; Cressie, 1993),

\[
\lambda(x) = \lim_{|dx| \to 0} \left\{ \frac{E[N(dx)]}{|dx|} \right\}.
\]

To clarify the notation here employed (borrowed from Diggle, 2003), \(N(A)\) denotes the number of points in a particular planar area \(A\), \(|A|\) is the surface of \(A\) and \(dx\) is an infinitesimal area containing the point \(x\). Therefore, intuitively, \(\lambda(x)dx\) expresses the probability that an event locates in an infinitesimal region centred at point \(x\) and with a surface area \(dx\) (Diggle et al., 2007).

For a stationary process, the first-order intensity is constant across all the area, that is \(\lambda(x) = \lambda\) for each \(x\), and then it represents the expected number of events per unitary area (Diggle, 2003).

The \textit{second-order} properties are described by a \textit{second-order intensity function} (Diggle, 2003; Cressie, 1993),

\[
\lambda_2(x, y) = \lim_{|dx|, |dy| \to 0} \left\{ \frac{E[N(dx)N(dy)]}{|dx||dy|} \right\},
\]
where \( x \) and \( y \) denote the coordinates of two distinct arbitrary points. Intuitively, \( \lambda_2(x, y)dx\,dy \) expresses the probability that two points locate in two infinitesimal regions centred in \( x \) and \( y \) and with surface areas \( dx \) and \( dy \) respectively (Diggle et al., 2007). Therefore, \( \lambda_2(x, y) \) characterizes the expected additional events located in \( y \) relative to a given event located in \( x \) and hence it represents the intensity of spatial interactions amongst points.

If a process is stationary we have that \( \lambda_2(x, y) \equiv \lambda_2(x-y) \); furthermore, if a process is stationary and also isotropic, \( \lambda_2(x, y) \) depends only on the Euclidean distance between \( x \) and \( y \), \( d = \|x-y\| \), and hence \( \lambda_2(x, y) = \lambda_2(d) \) (Diggle, 2003).

For a process characterized by independence amongst points’ locations, \( \lambda_2(x, y) = \lambda(x)\lambda(y) \). As a result, the function \( g(x, y) = \lambda_2(x, y)/\lambda(x)\lambda(y) \), known as pair correlation function (Ripley 1976, 1977), gives indications about the entity of dependence. In particular, if there is no spatial interaction between points of the process at locations \( x \) and \( y \) then \( g(x-y) = 1 \). On the other hand, when \( g(x-y) > 1 \) the process is characterized by positive dependence, meaning that points at locations \( x \) and \( y \) tend to attract each others, while if \( g(x-y) < 1 \) the process is characterized by negative dependence, which implies that points tend to repulse each others (Møller and Waagepetersen, 2007). Again, under the assumption of stationarity and isotropy, \( g(x-y) = g(d) \).

For a stationary and isotropic spatial point process, its second order properties can also be described by a different kind of function, introduced by Ripley (1976), which can heuristically be defined as follows:

\[
K(d) = \lambda^{-1}E\{\text{number of further points falling at a distance } \leq d \text{ from an arbitrary point}\}.
\]

Therefore, \( \lambda K(d) \) can be interpreted as the expected number of further points up to a distance \( d \) of an arbitrary point of the process (Ripley, 1977).

Under the further assumption that the process is orderly, which implies that each location cannot host more than one point, Ripley (1976) has shown that the link between \( K(d) \) and \( \lambda_2(d) \) is the following:

\[
\lambda K(d) = 2\pi\lambda^{-1} \int_0^d \lambda_2(u) du,
\]

or inversely,

\[
\lambda_2(d) = \lambda^2 (2\pi d)^{-1} K'(d).
\]

The link between the two function lies in the fact that both describe the distribution of the distances between pairs of points in a point pattern, where \( K(d) \) is related to the cumulative distribution function and \( \lambda_2(d) \) to the probability density function.

Since \( K(d) \) can more easily be estimated on data respect to \( \lambda_2(d) \), it has become the most popular empirical tool to detect the presence of spatial dependence, and then clustering, in a spatial point pattern. The estimator of \( K(d) \) will be illustrated in the Essays.

Three classes of spatial point processes can be relevant for the analysis of spatial clustering of economic activities: the homogeneous Poisson process, the inhomogeneous Poisson processes and the Poisson cluster processes.

### 1.2 The homogeneous Poisson process
The homogeneous Poisson process represents an idealized standard of the hypothesis of *Complete Spatial Randomness* (CSR). The postulates that define this stochastic generating mechanism, indeed, are equivalent to the definition of CSR (Diggle, 2003), which considering an hypothetical study region $A$ with a surface area of $|A|$, states:

(i) for any constant $\lambda > 0$, the $n$ number of points located in $A$ follows a Poisson distribution with mean $\lambda|A|$;

(ii) the $n$ points in $A$ constitute an independent random sample from the uniform distribution on $A$.

The parameter $\lambda$ of the process defines the first-order intensity of the resulting point pattern, that is the average number of points per unitary area. Therefore, condition (i) implies that the intensity of the study region $A$ is constant, and hence it corresponds exactly to the uniformity condition of CSR hypothesis.

On the other hand, condition (ii), which parallels the independence condition of CSR hypothesis, entails that the location of any point is independent from the locations of all other points. As a result, the second-intensity is given as

$$\lambda_2(d) = \lambda^2: d > 0,$$

which, according to equation (1), implies that

$$K(d) = \pi d^2: d > 0.$$

A partial realization of the homogeneous Poisson process can be obtained following a computational procedure working out in two steps. First of all, the simulation of the $n$ number of points from the Poisson distribution with mean proportional to the chosen value of $\lambda$ is required. Secondly, once the random value $n$ is returned, the $n$ events are generated independently according to a uniform distribution on the chosen study region.

Figure 2 shows two possible realizations of the homogeneous Poisson process on the unit square, with $\lambda$ parameter equal to, respectively, 50 and 100.

**Figure 2:** (a) A realization of a homogeneous Poisson process with intensity 50 in unit square; (b) A realization of a homogeneous Poisson process with intensity 100 in unit square
Alternatively, instead of referring to a random value of \( n \), we might be interested in implementing simulations conditional on a fixed number of points. As it can be seen in the three Essays, the conditioned simulated patterns have a crucial importance within the testing scope, because they can represent useful hypothetic theoretical counterfactuals of observed patterns.

In terms of computation, the conditioned simulations can be implemented working out the second step of the procedure above mentioned with \( n \) as a chosen constant. By way of illustration, Figure 3 shows two possible partial realizations of a homogeneous Poisson process conditional on 100 points on the unit square.

**Figure 3:** Two realizations of a homogeneous Poisson process conditional on 100 points in unit square

The homogeneous Poisson process constitutes a benchmark to understand the general characteristics of an observed spatial point pattern. Indeed, making use of simulated homogeneous Poisson processes conditional on the number of points of a reference observed pattern, we are able to run tests of randomness which allow to identify whether the situation under study belongs to the class of *aggregated* patterns (as Figure 1a), *regular* patterns (as Figure 1b) or *random* patterns (as Figure 1b). Once the property of non-randomness has been evaluated, the analyst is in the position to build models for non-random processes, and to test them to see whether they are suitable for representing reality or not.

1.3 *The aggregated processes*

The main violation of a CSR point pattern is represented by the *aggregated* point pattern, which may be of two types: the aggregated pattern caused by the “true contagion” of one point by another, and the aggregated pattern caused by “apparent contagion” between points (Arbia and Espa, 1996). In the context of the analysis of spatial clustering of economic activities, *apparent contagion* arises when exogenous factors lead to the location of firms in certain specific geographical zones. For instance, firms may cluster locally in order to exploit favourable conditions within the area, such as the presence of useful infrastructures, the proximity to communication routes or the possibility of benefiting from public incentives by locating in specific areas outside the residential centres.

On the other hand, *true contagion* occurs when the presence of one event in a given area stimulates the presence of other events nearby. For instance, the presence of “leader” firms encourages the settlement of “followers” in the same area, just as the incidence of knowledge spillovers accelerates industrial agglomerations.

From a statistical methodological point of view, apparent contagion is related to the violation of the CSR condition of *uniformity*; while true contagion to that of *independence*. Aggregated point patterns can then be generated by two distinct classes of stochastic mechanisms: the inhomogeneous...
Poisson processes, driving apparent contagion; and the and the Poisson cluster processes, driving true contagion.

1.3.1 Apparent contagion: the inhomogeneous Poisson processes

Apparent contagion derives from abandoning the hypothesis of uniformity of the homogeneous Poisson process. In this case, the point distribution is such that the intensity $\lambda$ is no longer constant throughout the territory. It may be higher in certain sub-regions of the area, and lower in others. As a consequence, there will be zones with a high intensity of points, and others with a low intensity of points and this will produce an aggregated pattern.

Apparent contagion will thus occur as a result of the fact that although each point is arranged in a random fashion, independent of the other points, the presence of some zones more suited to accommodating points than others leads to agglomerations (Arbia and Espa, 1996).

Therefore, in order to generate stochastically this kind of aggregated patterns, we replace the constant intensity $\lambda$ of the homogeneous Poisson process by an intensity function varying on the space, $\lambda(x)$ (Diggle, 2003). This procedure constitutes the class of inhomogeneous Poisson processes which, considering an hypothetical study region $A$ with a surface area of $|A|$, is characterized by the following postulates:

(i) the $n$ number of points located in $A$ follows a Poisson distribution with mean $\int_A \lambda(x)dx$;

(ii) the $n$ points in $A$ constitute an independent random sample from the distribution on $A$ with probability density function proportional to $\lambda(x)$.

A useful computational algorithm to simulate an inhomogeneous Poisson process was suggested by Lewis and Shedler (1979). It is based on ‘thinning’, that is: first of all it generates a homogeneous Poisson process of intensity $\lambda_0$ equal to the maximum value of the function $\lambda(x)$ on the study region $A$; then it deletes each point, independently of other points, with deletion probability $\lambda(x)/\lambda_0$.

By way of example, Figure 4 shows two partial realizations of an inhomogeneous Poisson process on the unit square, with spatially varying intensity function $\lambda(x) = 100 \exp(-3x_1)$ reaching a maximum value of 100.

**Figure 4:** Two realizations of an inhomogeneous Poisson process in unit square with intensity $\lambda(x) = 100 \exp(-3x_1)$ bounded by 100.
If we are interested in simulations conditional on a fixed number of points \( n \), we can follow the same algorithm. Specifically, spatial points are generated from the uniform distribution on \( A \). Then each generated point \( x = (x_1, x_2) \) is accepted with probability \( \frac{\lambda(x)}{\lambda_0} \) and rejected otherwise. The algorithm continues until \( n \) points have been accepted. Figure 5 shows two realizations of a conditioned inhomogeneous Poisson process, with 100 points, in which \( \lambda(x_1, x_2) = x_1^2 + x_2^2 \).

**Figure 5:** Two realizations of an inhomogeneous Poisson process conditional on 100 points in unit square with intensity \( \lambda(x_1, x_2) = x_1^2 + x_2^2 \).

### 1.3.2 True contagion: the Poisson cluster processes

True contagion results from relaxing the hypothesis of independence between the locations of points. Poisson cluster processes have been introduced by Neyman and Scott (1958) as a class of mechanisms that allow to model dependence among points and hence to incorporate an explicit form of spatial clustering (Diggle, 2003).

An interesting way of visualizing the genesis of an aggregated pattern by the means of a Poisson cluster process is the leader-follower scheme proposed in Arbia and Espa (1996).

Take an area and randomly allocate a certain number of leader firms within its boundaries. Then establish a threshold distance for local allied activities and, basing on this threshold, let us define an area of influence for each leader. In a first approximation this area may be considered to be the same for all leader firms. Now fix a certain number of followers for each leader, as the realization of a random variable. There are a number of examples in the literature where logarithmic or Poisson distribution were used (Upton and Fingleton, 1985).

Finally, allocate the followers to the leader firms on the basis of a given bivariate distribution. For example, a bivariate uniform distribution may be used in those cases where the probability within the area of influence can be taken to be constant or, alternatively, we may consider a bivariate Normal distribution if we assume that the probability of localising the followers decreases exponentially with an increase in the distance from the leader.

The resulting process is the Poisson cluster process shown in Figure 6 (borrowed from Arbia and Espa, 1996).
Figure 6: The genesis of a Poisson cluster process. (a) The allocation of leaders; (b) setting the dimensions of the areas of local allied activities and allocation of the followers; (c) the resulting point process.

The generation of a Poisson cluster process then consists of the following stages:

(i) a given number of leader points is generated over the surface of the study region so as to create a CSR pattern; in other words, the spatial positions of the leader points constitute an independent random sample from the uniform distribution on the study region;

(ii) each leader point produces a random number of followers, independently and identically generated for each leader according to a given probability distribution (e.g., Poisson, logarithmic);

(iii) the locations of the followers, with regard to the leader points that produced them, are located independently and identically on the basis of a given bivariate probability distribution.

The locations of the followers, for instance, might be uniformly distributed in a circle of a given radius or they may follow a radially symmetric Normal distribution with probability density function, such as

\[
h(x_1, x_2) = \left(2\pi\sigma^2\right)^{-1} \exp\left[-\frac{\left(x_1^2 + x_2^2\right)}{2\sigma^2}\right]
\]

where \((x_1, x_2)\) are the geographic coordinates of a follower point, and \(\sigma\) is a parameter to be established representing, in fact, the maximum spatial extension of the area of influence for each leader.

Figure 7 shows two realizations of a Poisson cluster process on the unit square characterized by intensity of the leader homogeneous process equal to 25 and 4 as the average number of followers per leader. In Figure 7(a), the location of each follower relative to its leader is realized following the uniform distribution on a random circular disc with a maximum radius of 0.025. On the other hand, in (b), such position follows the distribution represented by equation (2) with \(\sigma = 0.025\).
**Figure 7:** Two realizations of a Poisson cluster process in unit square with *leader* intensity 25 and expected number of *followers* per *leader* 4: (a) uniform dispersion of *followers*, with maximum radius 0.025; (b) radially symmetric Normal dispersion of *followers*, with $\sigma = 0.025$

In Figure 7(a), it can be clearly seen that the allocation probability of *followers* within the cluster is constant in all the area of influence. In contrast, the pattern represented by Figure 7(b) evidences that the *followers* intensity tends to diminish as the distance from the cluster centre increases.

Conditioning on the number of *leader* points and number of *followers* per *leader* is straightforward. Rather than establishing the parameters of random variables, we simply need to fix constant values. By way of illustration, Figure 8 shows realizations of a Poisson cluster process conditioned on fixed quantities.

**Figure 8:** Two realizations of a Poisson cluster process in unit square with 25 *leader* points and 4 *followers* per *leader*: (a) uniform dispersion of *followers*, with maximum radius 0.025; (b) radially symmetric Normal dispersion of *followers*, with $\sigma = 0.025$

In the left map of Figure 8 the 25 groups of 4 *followers* are clearly identifiable. Such a visual identification is not so straightforward in Fig. 18(b), due to the coalescence between nearby clusters.
For testing purposes, in order to recreate specific paradigmatic situations we might be interested in performing simulations conditional on the total number of events on the study region. Indeed, when the number of followers per leader is randomly generated according to a Poisson distribution, the point process can be simulated by randomly allocating a fixed number of events amongst the leader points.

This is illustrated in Figure 9 which shows two realizations wherein the 100 followers are randomly distributed amongst the 25 leader points.

**Figure 9:** Two realizations of a Poisson cluster process in unit square with 100 followers randomly allocated amongst 25 leader points and (a) uniform dispersion of followers, with maximum radius 0.025; (b) radially symmetric Normal dispersion of followers, with \( \sigma = 0.025 \). Average number of followers per leader equal to 4.

1.4 Cox processes

Another class of point processes which may be useful for economic analysis application is represented by the Cox processes. They represent a natural extension of the inhomogeneous Poisson processes where the source of spatial inhomogeneity, that is the intensity function \( \lambda(x) \), rather than being deterministic, is stochastically driven by a random process. Therefore, these processes are “doubly stochastic” (Cox, 1955; Grandell, 1976; Daley and Vere-Jones, 2003) and allow to explicitly model spatial intensity endogenously rather than exogenously. Formally, following Diggle (2003), a Cox process can be defined as follows,

(i) \( \{ \Lambda(x) = \lambda(x) : x \in \mathbb{R}^2 \} \) is a non-negative random field.

(ii) conditional on \( \{ \Lambda(x) = \lambda(x) : x \in \mathbb{R}^2 \} \), points are generated following an inhomogeneous Poisson process with intensity function \( \lambda(x) \).

The resulting point process is stationary if and only if the intensity random field is stationary itself (Diggle, 2003). In the stationary case, the first-order and second-order intensity functions are, respectively, given as

\[
\lambda = E[\Lambda(x)] \quad \text{and} \quad \lambda_2(x, y) = E[\Lambda(x), \Lambda(y)].
\]
Figure 10 shows a partial realization of a Cox process in which $\Lambda(x)$ is a Normal random field with mean $\mu = 100$, variance $\sigma^2 = 0.25$ and correlation function $\rho(d) = \exp\{-d/0.25\}$. Also shown is the underlying intensity surface $\lambda(x) = \Lambda(x)$ as a grey-scale image, where lighter are the grey colours higher is the value of intensity.

**Figure 10:** A realization of a Normal Cox process in unit square with mean $\mu = 100$, variance $\sigma^2 = 0.25$ and correlation function $\rho(d) = \exp\{-d/0.25\}$. (a) The generated underlying intensity surface (grey-scale image); (b) the generated point pattern.

---

2 Spatial point pattern statistics in economics

Spatial point pattern statistics has recently been used in the economic field in order to develop measures of spatial clustering comparable across spatial scales and unbiased with respect to arbitrary changes to spatial classification (Properties iv and v of an ideal measure outlined in the previous chapter).

The most advanced contributions in the literature are probably those of Duranton and Overman (2005), Arbia et al. (2008) and Marcon and Puech (2010). All this works consider the spatial distribution of economic activities as a spatial point pattern and regard the estimated second-order intensity of the pattern as a measure of spatial clustering. In particular, Arbia et al. (2008) use Ripley’s function, $K(d)$, and Marcon and Puech (2010) use a modified version of it, thus referring to the cumulative distribution of the distances between pairs of points. On the other hand, Duranton and Overman (2005) use a slightly modified version of the pair correlation function, $g(r)$, which instead is related to the probability density function of the distances between pairs of points.

In order to control for industrial concentration (Property iii), in all quoted papers departures from spatial randomness implying presence of spatial clustering have been detected by comparing the values assumed by the employed measure for the pattern of a given industry to those assumed for the pattern of the whole economy. In other words, these authors have developed measures of relative spatial concentration. Although relative measures are very useful in controlling for the idiosyncratic characteristics of the territories under study, on the other hand they do not allow direct comparisons across different economies (see Haaland et al., 1999 and Mori et al., 2005 for a more detailed discussion). The original contributions of this thesis in the three Essays explore the possibility to develop measures of absolute spatial concentration.
References


Essay 1

Detecting the existence of space-time clustering of firms
Detecting the existence of space-time clustering of firms

Abstract: The use of the $K$-functions (Ripley, 1977) has become recently popular in the analysis of the spatial pattern of firms. It was first introduced in the economic literature by Arbia and Espa (1996) and then popularized by Marcon and Puech (2003), Quah and Simpson (2003), Duranton and Overman (2005) and Arbia et al. (2008). All this researches have followed a static approach, disregarding the time dimension. Temporal dynamics, on the other hand, play a crucial role in understanding the economic and social phenomena, particularly when referring to the analysis of the individual choices leading to the observed clusters of economic activities. With respect to the contributions previously appeared in the literature, this paper uncovers the process of firm demography by studying the dynamics of localization through space-time $K$-functions. The empirical part of the paper will focus on the study of the long run localization of firms in the area of Rome (Italy), by concentrating on the ICT sector data collected by the Italian Industrial Union in the period 1920-2005.

Keywords: Agglomeration, Non-parametric measures; Space-time $K$-functions, Spatial clusters, Spatial econometrics.

JEL classification codes: C21 · D92 · L60 · O18 · R12

1. Introduction: The spatial-temporal analysis of clusters of firms

There is no question that the process of localization of firms in space is essentially a dynamic phenomenon. At the hearth of the observed spatial patterns of clustering (where firms tend to attract each other) or inhibition (where firms tend conversely to repulse each other), we always find considerations related to time, dynamics, lagged dependence and evolution. As a matter of fact we cannot study phenomena like firm demography, birth-death processes and growth in space disregarding the time dimension. Yet in the literature the study of the clustering of firms in space and time have stubbornly followed two separated histories. On one side there is a long tradition of a substantial number of techniques available for modelling clustering of firms in time based on purely time-series methods and on the analysis of business cycles (Hamilton, 1994). These techniques may assist in the identification of situations of time-concentration where we observe a higher number of new firms in some particular periods due to cyclical movements or trends. On the other side research on spatial clustering of economic activities has only a more recent history and it is originated by a reinterpretation of Marshall's insights on 19th-century industrial localization operated by some authors in the nineties (e.g. Krugman, 1991; Fujita et al., 1999). Following these seminal works the empirical analysis of spatial clusters has developed along two distinct lines of research. The first was an attempt to examine directly the underlying economic mechanism, using the spatial dimension only as a source of data (see e.g. Ciccone and Hall, 1996; Jaffee et al., 1993; Rauch, 1993; Henderson, 2003). The basic methodology here is that of a panel data or pure spatial regressions that employs observable covariates related to space (Arbia, 2006; Baltagi, 2008). The second line of researches attempts to characterize the spatial distribution of economic activities by observing the joint behaviour of the different units distributed across space (Devereux et al., 2004; Duranton and Overman, 2005; Ellison and Glaeser, 1997; Ioannides and Overman, 2004). The reference methodology under this respect is that of the spatial point pattern analysis (Diggle, 2003). In particular in this field the use of the $K$-functions (Ripley, 1977) has become recently popular.
First introduced in the economic literature by Arbia and Espa (1996) was then popularized by Marcon and Puech (2003), Quah and Simpson (2003), Duranton and Overman (2005) and Arbia et al. (2008). In particular Arbia et al. (2008) proposed the use of Ripley’s $K$-functions as an instrument to study the inter-sectoral co-agglomeration pattern of firms in a single moment of time.

The analysis of clusters in space and time, thus, have followed so far two different roads and two separated methodologies with no interactions among them. Time series methods have generally disregarded the spatial dimension while spatial clustering models have been essentially static and they analysed just the outcome of the dynamic adjustments as it is observed in one single moment of time. This approach is obviously partial and doomed to leave without explanations a number of different empirical cases that may occur in practice. In fact new firms settlements may display no spatial concentration if we look separately at each moment of time and yet they may present a remarkable agglomeration if we look at the overall resulting spatial distribution after a certain time period.

The importance of taking into account the temporal dynamics when analyzing spatial patterns of events has been well explained by Getis (1964) and Getis and Boots (1978). In particular, in this second work, referring to a straightforward “framework for viewing spatial processes”, they argue that without knowing the temporal evolution of the phenomenon under study it is not possible to identify the mechanism generating its spatial structure. In particular, they show that different space-time processes can lead to resulting spatial patterns which look the same. As a consequence, only phenomena which exhibit no increase or decrease of points over time might be represented as pure spatial processes (Getis and Boots, 1978) and hence could be meaningfully analysed neglecting the time dimension.

With respect to the contributions previously appeared in the literature, this paper attempts to unify the two approaches and to uncover the process of firm demography in a more comprehensive way by tackling it, both under a spatial and under a temporal point of view, within a unified framework. This framework is provided by the theory of space-time $K$-functions born as an extension of the simple synchronic $K$-function (introduced ago by Ripley, 1976 more than 30 years ago) and of the so called second-order analysis of point patterns (see, e.g. Getis and Boots, 1978). Examples of applications may be found in the regional science (Feser and Sweeney, 2000 and 2002), geography (Getis, 1983; Okabe et al., 1995; Yamada and Rogerson, 2003) and ecology literature (Goreaud and Pelissier, 1999 and 2003; Haase, 2003).

In the epidemiological context, Diggle et al. (1995) have proposed an extension of the spatial univariate $K$-function to allow for the detection of space-time interactions in what was termed a time-labelled spatial point pattern. Our purpose is to introduce this statistical framework in the context of economic geography to study the interactions between the spatial and temporal distributions of firms. Specifically, we intend to test empirically the presence of space-time clustering of firms. Once the significance of space-time clustering phenomenon is assessed by using the space-time $K$-function approach, we will be in the position to test the presence of hypothetical spatial configurations like, e.g., leader-follower patterns or the presence of spatial segregation between ‘old’ and ‘young’ industries.

In order to assess the scientific scope of our proposed methodology we need to clarify preliminarily what we mean with “spatial cluster of firms”. The notion of spatial concentration we refer in the present context is the “topographic concentration”, as defined by Brühlhart and Traeger (2005), which evaluates the geographic distribution of economic activities only relative to physical space. In such a context, the absence of concentration benchmark is represented by a spatial pattern where the firms are randomly spread over the physical space. Therefore, the departures from this random spatial diffusion are considered as clusters without taking into account the spatial distribution of exogenous variables like traffic accessibility, factor endowments, skills and labor force potential. In other words, in the present context, the spatial component of the space-time clustering phenomenon can be jointly determined by the interactions among economic agents – driven, for example, by the presence of knowledge spillovers or external economies of scale – and
the exogenous features of the territory (such as the presence of useful infrastructure, the proximity to communication routes or the possibility to benefit from public incentives to locate in specific areas outside the residential centres). However, the firms that we refer in the case study considered in Section 3 belong to the ICT sector and (unlike, e.g., the agricultural and heavy manufacturing industries) they do not need a particularly weighty endowments. Therefore, we can reasonably argue that the first factor, rather than spatial heterogeneity, is prevalent in driving clusters.

The structure of the paper is the following. In Section 2 we will introduce the methodological framework and we will present the theory of the space-time $K$-functions. To help the interpretation of the subsequent empirical analysis, in this Section we will also describe some stylized spatial distributions of firms that may occur in empirical cases and the corresponding behaviour of the $K$-functions diagnostics. Section 3 will be devoted to the empirical part of the paper by first introducing the working dataset based on the spatial distribution of Information Technology and Communication (ICT) firms in the area of Rome (Italy) collected by the Industrial Union in the period 1920-2005. It will also contain the empirical application of the models presented in Section 2 based on this dataset. Section 4 contains the discussion of the results and the analysis of their economic implications. Finally Section 5 contains some concluding remarks and directions for future developments in this field.

2. The statistical methodological framework

2.1 Space-time $K$-function analysis

Economic events, such as the establishment of new firms, may occur at different points in space and time. As a consequence, in order to study the geographic concentration of industries, we should control for the temporal dynamics that characterize the localization processes. Accordingly, we need to explore the possibility that the spatial and temporal phenomena, producing the observed pattern of firms at a given moment of time, interact to provide space-time clustering. This requirement can be performed referring to a statistical test about the independence between the spatial and the temporal distribution of firms. In the case of dependence, the geographic pattern of firms is characterized by the presence of space-time interaction meaning that such a pattern cannot be explained only by static factors, but we should also consider the dynamic evolution of the spatial concentration phenomenon.

Univariate spatial $K$-functions (proposed by Ripley 1976 and 1977) have been already used in the economic literature to detect the geographical concentration of industries (see e.g., Arbia and Espa 1996; Marcon and Puech 2003; Quah and Simpson 2003). They can be exploited in a dynamic context by analysing separately the spatial and the temporal clustering pattern. However, a more comprehensive approach refers to the analysis of both dimensions simultaneously thus paying attention also to the space-time interactions. In this paper we will consider a dynamic extension of the univariate $K$-functions proposed and fully described in Diggle et al. (1995). In what follows we will present a brief account of the theory of space-time $K$-functions. The symbolism and definitions are in accordance with those used in Arbia et al. (2008) to which the reader is referred for the simple, purely spatial, $K$-function.

Generally speaking, the technique involves the comparison between the observed spatio-temporal point pattern and a theoretical pattern that has the same temporal and spatial properties as the original data, but no space-time interaction (Diggle et al., 1995; French et al., 2005). In this context an auxiliary information is associated to every observed spatial point in the form of the time of occurrence. Under the assumption of stationarity and isotropy (Diggle, 2003; Arbia, 2006), we can build up the space-time $K$-function:

\[ \lambda_{dt} K(d,t) = E \{ \# \text{ of points falling at a distance } d \text{ and a time } t \} \leq \]
(French et al. 2005), with $E\{\cdot\}$ indicating the expectation operator and the parameter $\lambda_{DT}$ representing the spatial and temporal joint intensity of the point process, i.e. the number of points per unitary area and per unit time. If the processes working in time and space are independent (that is if there is no space-time interaction) the functional $K(d,t)$ should be equal to the product of the spatial and temporal K-functions $K_D(d)K_T(t)$ (Diggle et al., 1995), where $K_D(d)$ and $K_T(t)$, are defined, respectively, as follows:

$$\lambda_D K_D(d) = E\{\# \text{ of points falling at a spatial distance} \leq d \text{ from an arbitrary point}\}$$ (2)

and

$$\lambda_T K_T(t) = E\{\# \text{ of points falling at a time interval} \leq t \text{ from an arbitrary point}\}.$$

In the previous expressions $\lambda_D$ represents the spatial intensity, that is the number of points per unitary area. In a similar fashion $\lambda_T$ denotes the temporal intensity, i.e. the number of points per unit time. The meaning of a univariate spatial K-function ($K_D(d)$) is well known (see Ripley, 1977 and Arbia et al., 2008 for economic interpretation). They suggest visually if the firms tend to concentrate significantly in some portions of the study area rather than in others. Purely temporal K-function ($K_T(t)$) are not treated in the statistical literature, but could be used to identify if in the observed time series a significantly higher number of firms is concentrated in some periods rather than in others.

In the case of no space-time interaction, we might theoretically expect that $K(d,t)=K_D(d)K_T(t)$ (Diggle et al., 1995; Gatrell et al., 1996). The product functional $K_D(d)K_T(t)$, in fact, represents the expected K-function under the hypothesis of absence of space-time interaction and can be used as a reference for comparison with the observed space-time K-function, $K(d,t)$.

Turning now to the estimation aspects, considering a univariate ‘time marked’ point map, we can define the estimators of the three component processes (i.e. $K(d,t)$, $K_D(d)$ and $K_T(t)$) by close analogy to those suggested in the unmarked univariate case (Ripley, 1977; Diggle, 2003).

To start with, let us consider the space-time K-function which, as we mentioned above, is represented by the expected number of points within a spatial distance $d$ and a time interval $t$ of an arbitrary point, scaled by the expected number of points per unitary area and per unit time. Diggle et al. (1995) have shown that a proper edge-corrected estimator of $K(d,t)$ from an observed ‘time marked’ point pattern with $n$ observations can be the following:

$$\hat{K}(d,t) = \frac{AT}{n^2} \sum_i \sum_{j \neq i} I_d(d_{ij})I_t(t_{ij}) w_{ij} v_{ij}$$

where $A$ is the total surface of the area and $T$ is the whole observed interval of time. In addition the terms $d_{ij}$ and $t_{ij}$ represent, respectively, the spatial distance and the time interval between the $i$th and $j$th observed points. Finally $I_d(d_{ij})$ and $I_t(t_{ij})$ represent indicator functions assuming the value 1 if $d_{ij} \leq d$ and $t_{ij} \leq t$, respectively, and 0 otherwise.

Due to the presence of spatial and temporal edge effects (which might potentially distort the estimates close to the boundary of the area $A$ and to the time limits of $T$) the adjustment factors $w_{ij}$
and \(v_{ij}\) are introduced. The weight function \(w_{ij}\) expresses the proportion of the circumference of a circle centred on point \(i\), passing through the point \(j\), which lies within \(A\) (Boots and Getis, 1988). By analogy, the factor \(v_{ij}\) refers to the time segment centred on \(i\), of length \(t_{ij}\), lying within the observed total duration time, between 0 and \(T\) (Diggle, 2003; Diggle et al., 1995; Gatrell et al., 1996).

Referring to the same statistical framework, the edge-corrected estimators of the spatial and temporal \(K\)-functions, \(K_D(d)\) and \(K_T(t)\) are defined, respectively, as:

\[
\hat{K}_D(d) = \frac{A}{n^2} \sum_i \sum_{j \neq i} \frac{I_d(d_{ij})}{w_{ij}}
\]

(Boots and Getis, 1988; Diggle, 2003) and:

\[
\hat{K}_T(t) = \frac{T}{n^2} \sum_i \sum_{j \neq i} \frac{I_t(t_{ij})}{v_{ij}}
\]

(Diggle et al., 1995; Bailey and Gatrell, 1995). As already said, when there is no space-time interaction we have that \(K(d,t) = K_D(d)K_T(t)\). As a consequence one possible exploratory tool for the independence between the processes operating in time and space is the functional:

\[
\hat{D}(d,t) = \hat{K}(d,t) - \hat{K}_D(d)\hat{K}_T(t)
\]

(Gatrell et al., 1996). This functional is proportional to the increased numbers of points within spatial distance \(d\) and time interval \(t\) with respect to a process which possesses the same temporal and spatial characteristics, but no space-time interaction. As a consequence, the presence of space-time interactions might be revealed in the appearance of peaks on the 3-dimensional surface of \(\hat{D}(d,t)\) plotted against the spatial distance and the time sequence.

Diggle et al. (1995) and French et al. (2005) have proposed a transformation of (3) which allows for the possibility of working with relative quantities rather than absolute numbers. This is defined as:

\[
\hat{D}_0(d,t) = \hat{D}(d,t) / \{\hat{K}_D(d)\hat{K}_T(t)\}
\]

(4)

Expression (4), the “Diggle function”, is proportional to the relative increase in points within spatial distance \(d\) and time interval \(t\) with respect to a process with the same temporal and spatial characteristics, but no space-time interaction. Similarly to \(\hat{D}\), the functional \(\hat{D}_0\) can be plotted in a 3-dimensional graph versus \(d\) and \(t\) to help the visualization and the detection of interdependence between the spatial and temporal processes.

As we stated at the beginning of this section, the \(K\)-functions-based empirical method quantifies explicitly the spatial dependence between events under the working assumption of spatial homogeneity (or stationarity). This is expressed by the fact that in Equations (1) and (2) the spatio-temporal joint intensity and spatial intensity (\(\lambda_{DT}\) and \(\lambda_D\)) respectively, are assumed as constant. A natural way to overcome this procedural limit consists of allowing these two quantities to vary over space. We could express these two functions as \(\lambda_D(x)\) and \(\lambda_{DT}(x)\), where the argument \(x\) represents the geographic coordinates of an arbitrary point. Limiting to the mere spatial perspective, and hence neglecting the temporal evolution, in Arbia et al. (2009) we followed this approach to analyse the spatial interactions among firms of the high-tech industry in Milan, while controlling for the
exogenous effects of the characteristics of the study area. One of the primary tasks of the future research agenda in the subject will be that of extending this analytical procedure to the temporal perspective, in order to develop a method to assess the space-time clustering phenomenon in an inhomogeneous spatial and temporal environment.

2.2 Some stylized space-time distributions

Before moving to presenting the important aspects related to the inferential evaluation of space-time interaction, in this Section it is useful to present some stylized situations that may occur in empirical cases when observing the spatial distribution of firms. The exam of these extreme, paradigmatic, situations and the analysis of the corresponding behaviour of the $K$ functionals, will help the interpretation of the functionals $\hat{D}(d,t)$ and $\hat{D}_0(d,t)$ in the case study that will be analysed in Section 3.

Figure 1 reports some theoretical spatial distributions of firms and the corresponding diagnostic plots. These could be used as benchmarks to be compared with the empirical situations that may be observed in practical instances. In doing so we follow an approach that proved already useful in a study by Getis (1964) on the changes in the commercial land-use pattern. In the quoted paper the author derives a series of stylised spatial point patterns built under different economic hypotheses of birth, death and diffusion, and then tries to identify the pattern that is closer to the observed distribution.

Of course those depicted in Figure 1 are but a few examples and certainly they do not exhaust all the cases that may be found in practice. We need to clarify that the distributions reported in column a) of the figure are stylized simplified arrangements based on only few points and are is used just to clarify the five extreme situations. In contrast the graphs reported in columns b) and c) are not based on the same points displayed in column a) (that were not sufficient to interpolate meaningfully the 3-D graphs) and are obtained through simulations based on a much larger number of points. Notice that the time dynamics in these examples is partially masked by the fact that we do not consider the death of firms, but only the process of new firm creation.

Case i) refers to the instance of clustering in space with no clustering in time and no space-time interaction. The map appears with a strong visual impression of clustering in each time period, but the number of newly born firms is constant over time (see the different time symbols in the maps), thus displaying no time concentration. The situation is represented by a flat $\hat{D}_0(d,t)$ function in the time direction and a peak at distance 0.06 in space. Case ii) refers to the opposite situation where we do have clustering in time, but no clustering in space and no interaction. In this case the visual impression is that of spatial randomness both in each time period taken individually and as a whole, but the number of firms in some periods is significantly higher than in others with, in particular, a strong concentration of new firms in the first time period. This situation is revealed in the graph c) with a peak of the $\hat{D}_0(d,t)$ function in time at lag 2 and an (almost) flat function on the spatial axis. Case iii) refers to the case of no space, no time clustering and, as a result, no interaction and has an appearance of no clustering in space with new firms that are created randomly in the different time periods. The $\hat{D}_0(d,t)$ function here is flat in both the space and time direction. Case iv) considers the instance where points are clustered both in time and space with an interaction between the two dimensions. Graph a) presents points that are highly agglomerated in space if observed in each individual time period and also if we look at all points jointly disregarding the different time markers. In this graph it is also evident a strong time concentration with a higher number of new firms created in the second time period. Finally Case v) considers the situation where points are clustered both in time and space, but there is no interaction between the two dimensions. Observing each year individually produces a visual impression of clustering, however looking at the whole map without distinguishing between the
different time periods the visual impression is that of randomness. There is also a considerably high
degree of time concentration with a higher number of new firms created in the first time period.
This situation was generated artificially considering the product of the two marginal $K$-functions in
space and time separately.

It is important to stress that the purpose of the proposed methodology is to discriminate if the
observed space-time pattern of economic activities is driven by a systematic mechanism or purely
by chance. We argue that this is an important first step in the analysis of space-time dynamics of
firms. In fact, if the location pattern is primarily due to randomness, then any economic model
trying to explain the observed pattern would be meaningless. As a consequence we argue that the
use of the proposed tools is preliminary to the detection of the relevant economic factors that
determined the observed spatial configuration. This second important step, however, could only be
tackled with different tools and with a larger information set on structural variables other than just
the geographic location, such as the plant size, the characteristics of the local demand and the
workforce market potential. For these reasons this second step is not undertaken here and is left to a
future study.

Obviously any substantive conclusions on the prevailing spatial and time pattern cannot be based
merely on the visual inspection of the empirical graphs contrasted with the stylized pattern and they
need a more grounded validation based on sound inferential tools. These tools will be introduced in
the following Section.

2.3 Inference

In the present Section we will introduce an inferential framework in order to formally assess the
significance of the empirically observed values of $\hat{D}(d,t)$. However, since the exact distribution
of the functional $D$, is unknown, its variance cannot be evaluated theoretically and no exact statistical
testing procedure can be adopted. To overcome this aspect Diggle et al. (1995) suggested to obtain
a significance test by exploiting a Monte Carlo approach. In the quoted paper the authors suggested
to perform $m$ simulations, where at each step the $n$ geographical points are marked at random with
the observed $n$ time ‘markers’. Having thus obtained $m$ simulated spatial-temporal point patterns,
we can thus compute $m$ different estimates of $\hat{D}(d,t)$. We will refer to these estimates with the
symbol $\hat{D}_i(d,t), i = 1, \ldots, m$. The observed variance of these $m$ estimates, say $\hat{V}(d,t)$, can be
reasonably used as an estimator of the variance of $\hat{D}(d,t)$ (Gatrell et al., 1996). Having introduced
these definitions, we can also introduce the idea of “standardized residuals” as

$$\hat{R}(d,t) = \hat{D}(d,t) / \sqrt{\hat{V}(d,t)}.$$  (5)
Figure 1: Some theoretical spatial arrangements of firms in space and time (column a) and the corresponding $\hat{D}(d,t)$ (column b) and $\hat{D}_0(d,t)$ plots (column c). Space is represented by a unit square. Time is represented by the set $[1,4] \in N$.

<table>
<thead>
<tr>
<th>a) Spatial distribution</th>
<th>b) $\hat{D}(d,t)$ function</th>
<th>c) $\hat{D}_0(d,t)$ function</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Spatial distribution" /></td>
<td><img src="image2" alt="Function plot" /></td>
<td><img src="image3" alt="Function plot" /></td>
</tr>
<tr>
<td>Case i) $p$-value Test Monte Carlo: 0.443</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Case ii) $p$-value Test Monte Carlo: 0.104 |
| ![Spatial distribution](image4) | ![Function plot](image5) | ![Function plot](image6) |

| Case iii) $p$-value Test Monte Carlo: 0.477 |
| ![Spatial distribution](image7) | ![Function plot](image8) | ![Function plot](image9) |

| Case iv) $p$-value Test Monte Carlo: 0.001 |
| ![Spatial distribution](image10) | ![Function plot](image11) | ![Function plot](image12) |

| Case v) $p$-value Test Monte Carlo: 0.401 |
| ![Spatial distribution](image13) | ![Function plot](image14) | ![Function plot](image15) |

NOTES: Case i: Spatial clustering, no time clustering, no space-time interaction
Case ii: No spatial clustering, time clustering, no space-time interaction
Case iii: No spatial clustering, no time clustering, no space-time interaction.
Case iv: Spatial and time clustering and space-time interaction.
Case v: Spatial and time clustering, no space-time interaction.

For the meaning of the $p$-value Monte Carlo test see the discussion in Section 2.3. $p$-values larger than 0.05 refer to non significant space-time interaction. Figures a) are based on only few points to help the visualization. Figures b) and c) are based on a larger number of simulated points.
It is better to clarify that the term “standardized residuals” is the one used in the literature, as suggested by Diggle et al. (1995), and refers to the ratios between the observed values of \( \hat{D}(d,t) \) and its estimated standard deviation, reported in Equation (5). However, the use of this term may be misleading since it has nothing to do with the meaning more commonly assigned to it in regression analysis. In practice, it represents the excess number of points of \( \hat{K}(d,t) \) with respect to \( \hat{K}_D(d)\hat{K}_T(t) \), and it is a measure of space-time interaction. In the absence of any space-time interaction, these residuals have zero expectation and a variance equal to one. Therefore, an appropriate inferential method to test if the spatial and temporal processes are independent on one another consists in plotting the graph associated with Expression (5) against the product-functional \( \hat{R}(d,t) \). If there is no space-time interaction then approximately 95% of the values of \( \hat{R}(d,t) \) would lie within two standard errors (French et al., 2005). The interpretation of the \( \hat{R}(d,t) \) plot is not always straightforward. In fact, it could be masked by the fact that the residuals could be strongly dependent. In addition to this test, a further overall Monte Carlo testing procedure of space-time clustering has been suggested. It consists in taking the actual observed sum of the functionals \( \hat{D}(d,t) \) over all \( d \) and \( t \) and making a comparison with the empirical distribution of the \( m \) analogous sums of \( \hat{D}_i(d,t) \) over all \( d \) and \( t \), \( (i = 1,\ldots,m) \). A particularly high value of the observed sum among the values of this ‘artificial’ distribution would constitute evidence of overall space-time interaction. For example, as Gatrell et al. (1996) pointed out, if the observed sum is ranked above 95 out 100 simulated values, then the probability that the observed space-time interaction occurred by pure chance is less than 5 per cent.

3. Analysing the long run spatial dynamics of firms: the case of ICT industries in Rome (Italy) 1920-2005

3.1 Economic background: theoretical expectations for ICT firms location in cities

In the last years there was a flourishing of studies on the increase of a knowledge-based economy (OECD, 1996; 2001; Drucker, 1998; Foray, 2000; David and Foray, 2003; Cooke et al., 2007) and a number of spatial economic theories can be found in the literature on industrial agglomeration which can help in postulating the expected location patterns of ICT firms.

At the risk of oversimplifying the discussion, and without claiming to exhaust the vast literature on the subject, we can distinguish between at least two broad lines of thought. According to a first, consistent, part of the literature, one should expect the ICT firms to be spatially concentrated within the big metropolitan areas. Indeed, the idea of a strong connection between spatial clusters and economic performance where knowledge matters significantly has a very long tradition that can be traced back to the seminal contributions of Alfred Marshall (1920), and to the following re-appraisal of Perroux (1950), Hirschman (1958) and Jacobs (1961). Fundamentally, the expectation of clustering of ICT firms is supposed to be driven by the so-called tacit knowledge (Nonaka and Takeuchi, 1995; Polanyi, 1966) assuming the knowledge to be transferable only through direct face-to-face interaction (Storper and Venables, 2003). As a matter of fact, the new forms of technological knowledge are usually tacit, in the sense that their accessibility is bounded by geographic proximity of high-technology firms or knowledge institutions and by the nature and extent of the interactions among these actors in an innovation system (Lambooy and Van Oort, 2005). Therefore, the knowledge spillovers should be more easily picked up in cities, where many specialized workers are concentrated into a relatively small and limited space and where the transmission of new knowledge tends to occur more efficiently by direct human interaction (Glaeser et al. 1992; Henderson et al. 1995; Dumais et al. 2002; Van Oort.
The role of geographical and cultural proximity for tacit knowledge exchange has been discussed extensively in the literature on high-technology clusters (Saxenian, 1994; Storper, 1997, 2002; Porter, 1998; Keeble and Wilkinson, 2000; Yeung et al. 2007) and innovative milieux (Capello, 1999; Rallet and Torre, 1999).

In contrast, according to a second line of research primarily leaded by the works of Sassen (1994), Castells (1996) and Cairncross (1997 and 2001), we should expect the ICT firms to locate without a significant spatial structure. Essentially, these authors argue that the rapid development of the communication technologies in the 90’s (and the advent of internet above all) has limited the role of space in the locational choices of economic agents. As a consequence the exchange of knowledge and information is now less dependent on physical flows within the geographical space than it used to be in the past due to the increased possibility of communicating in real time with any point in the world.

Thus, the economic theory suggests at least two different location phenomena characterizing the ICT industry: one leading to spatial concentration, the other driving to geographic patterns where spatial interactions among economic agents are irrelevant.

In the following sections we will try to assess the empirical relevance of these two competing lines of thought by studying the spatial configuration of the ICT industries in the metropolitan area of Rome. In particular in Section 3.2 we will present the database and in Section 3.3 we will make use of the space-time methodology illustrated in Section 2 to test whether the spatial and temporal distributions are dependent and if there is any space-time interaction in the locational choices of ICT firms.

3.2 Data description

The empirical part of this paper focuses on a set of micro data on the firms of the ICT sector in the area of Rome. This dataset has been collected over a fairly long period ranging from 1920 to 2005 by the Industrial Union of Rome (UIR). The large time span considered could, in principle, create problems in the classification of industries that may have changed over time. However in the ICT sector, the firms that were still operating in 2005 could have changed their denomination, but not the typology of their product that remained constant over time. The dataset reports the full address and the year of establishment of the 169 industries currently operating in the area thus disregarding those that were born in the period considered, but that did not survive until the present year. The ICT industries are further classified into two groups: Electronic and communication (C) and Information Technology (IT). In our database there are 66 firms belonging to the first group and 103 belonging to the second group. Table 1 reports the time evolution of the sector in the 85 years considered in terms of the number of firms born in each decade. It is evident the slow development of the sector until the early eighties and the more pronounced increase in the birth of new companies in the eighties and in the nineties. The dynamic is very similar for the Electronic and Communication and the Information Technology sectors.

<table>
<thead>
<tr>
<th>Years of establishment</th>
<th>Number of firms</th>
<th>ICT = (1) + (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>1920-1960</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>1961-1970</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>1971-1980</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>1981-1990</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>1991-2000</td>
<td>29</td>
<td>37</td>
</tr>
<tr>
<td>2001-2005</td>
<td>8</td>
<td>25</td>
</tr>
<tr>
<td>Total</td>
<td>66</td>
<td>103</td>
</tr>
</tbody>
</table>
The spatial distribution of the 169 firms of the \textit{ICT} sector is reported in Figure 2, which also provides an idea of the dynamics by marking with different symbols firms born in the different decades. Data are available at the finest level of time and spatial resolution, having in hand the full address and the exact date of registration of them, and it is only for the purpose of illustration that they have been temporally grouped into decades in Table 1 and Figure 2.

**Figure 2**: Spatial location and time evolution of the location of 66 firms of \textit{ICT} sector in Rome (Italy), 1920-2005. Source: Our computations on the data provided by the UIR.

Figure 2 (a) reports the spatial distribution of the whole \textit{ICT} sector. It is clear from a first visual inspection of the graph that data display a marked tendency to cluster by concentrating in some specific portions of space, namely the central part of the area and particularly near the city centre.
Different graphical symbols are used to illustrate the temporal dynamics. However in Figure 2 (a) the number of points is too large to display any definite time pattern that cannot be easily identified on purely visual considerations and requires further and more sophisticated analysis. The tendency to cluster is rather different in the two groups. By looking at the different graphical symbols, in the case of Figure 2 (b), due to the limited number of points that are reported (only 66), we can also notice a certain tendency of firms to locate in space nearby industries existing in previous time periods thus revealing evidence of space-time interaction.

This preliminary descriptive analysis clearly indicates that the observed pattern is characterized by a distinctive spatially and temporally localized process, but with remarkably different features in the two groups of firms that need further investigation.

The visual inspection adds further scope to the space-time analysis in that it emphasizes the interest to test whether the spatial and temporal distributions are dependent and if there is any space-time interaction. This aim will be accomplished by the next Section.

3.3 Analysis of the K-functions

3.3.1 Analysing the distribution of the ICT sector

Figure 3 reports the plot of the space-time $K$-functions\(^1\) computed for the 169 firms of the whole ICT sector, as a function of both space and time. In particular, Figure 3(a) reports the absolute functional $\hat{D}(d,t)$ where the spatial distance ranges between 0 and 9 miles (9 being one forth of the maximum possible distance in the graph) while the temporal lag ranges between 0 and 21 (21 being one fourth of the time span that is 85 years). This limitation is due to the corrections that are needed in order to minimize the distortions induced by border effects (see Haase, 1995; Goreaud and Pélissier, 1999; Arbia et al., 2008).

**Figure 3:** 3-dimensional plot of the (a) $\hat{D}(d,t)$ function and (b) $\hat{D}_0(d,t)$ function for the ICT sector as a whole.

The exam of Figure 3(a) clearly suggests the presence of space-time clustering, but the extent of such phenomenon is not noticeable because of the range of computed values of $\hat{D}(d,t)$ is too narrow. In order to investigate more formally this space-time effect of interdependence, we also computed the relative functionals. Figure 3(b) reports the plot of $\hat{D}_0(d,t)$ (see Equation (4)). From

---

\(^1\) All the computation of the $K$-functions and the related analysis were implemented using the SPLANCS library (Rowlingson and Diggle, 1993) available in the R software.
the graph it is evident a peak at the short spatial distances (around the zero) and at a temporal interval of one and a second peak at a distance of approximately 1 mile and a time lag of 5 years. This shows that the underlying concentration phenomenon tends to drive clusters with a small spatial magnitude (circles with radius of 1 mile) and where the firms are temporally correlated in terms of the year of establishment.

To evaluate this result more formally under an inferential point of view, a set of 999 simulations was performed, permuting at random the time ‘markers’ attached to every point, thus allowing us to plot the standardized residuals against the product of the separate spatial and temporal $K$-functions (Figure 4). As we have mentioned in Section 2, in the case of no space-time interaction the standardized residuals are expected to have zero mean and a unitary variance. Figure 4 cannot be attached to any substantive economic interpretation. Indeed, it only constitutes a graphical inferential tool that can only be used to decide in favour of the presence or absence of a significant spatial structure in the observed pattern.

In the empirical case examined, we can clearly see that a relatively large number of estimated residuals lays above 2 standard errors (corresponding to 34.5% of cases), providing support to the hypothesis of interaction between the spatial and temporal component processes. However, because of the, potentially strong, interdependence amongst the estimates $\hat{R}(d,t)$ for different values of $d$ and $t$, this diagnostic plot cannot be considered particularly robust.

For this reason, in order to test the statistical significance of the results reported in Figure 4, a Monte Carlo test of space-time clustering was performed. Figure 5 displays the frequency distribution of the sum of the differences between the space-time $K$-function and the product of the separate space and time $K$-functions as they occurred in the 999 simulations. The sum of such differences in the observed dataset ranked 998 out of 1000. Therefore, the empirical $p$-value of the test is 0.002, thus providing formal evidence for the space-time clustering situation described by the plot of $\hat{D}_{0}(d,t)$. In other words, in the Rome area, the firms belonging to the ICT sector tend to agglomerate at a relatively small geographical distance and, moreover, the clusters are constituted by firms that established in the area with a strong dynamic component.

**Figure 4**: Plot of the estimated standardized residuals of $\hat{R}(d,t)$ against $\hat{K}_{D}(d)\hat{K}_{T}(t)$ for the ICT sector.

As already said the ICT sector is constituted by two groups, namely *Information Technology* and *Electronic and Communication*. In this Section we wish to analyse the concentration pattern and its dynamics of the two groups separately.
To start with, Figure 6 reports the plot of the absolute and relative $D$ functionals for the group of firms belonging to the *Information Technology* industries.

**Figure 5**: Empirical frequency distribution of the sum of the differences between the space-time $K$-function and the product of the separate space and time $K$-functions in 999 simulations. *ICT* sector.

**Figure 6**: 3-dimensional plot of the (a) $\hat{D}(d,t)$ function and (b) $\hat{D}_0(d,t)$ function for the *Information Technology* sector.

### 3.3.2 Disaggregated analysis of the two groups: “Information Technology” and “Electronic and Communication”

The visual features of this graph are rather different from those observed for the *ICT* sector as a whole (see Figure 3). In fact, the $\hat{D}_0(d,t)$ functional displays a rather less marked spatial clustering and a negative time cluster (graph below the zero line in the time direction). More specifically Figure 6(a) displays the plot of $\hat{D}(d,t)$ where, in order to manage the edge effects, the spatial distance and the temporal lag range, respectively, between 0 and 5 miles and 0 and 11 years. Although the graph evidences some peaks in the surface of the functional, their magnitude is small. Indeed, the higher positive peak reaches a value of 0.025 and analogously the lower negative extremity is −0.025. As a consequence, this does not support the hypothesis of significant space-time interaction.
On the other hand, the plot of the relative functional $\hat{D}_0(d,t)$ (Figure 6(b)) might suggest the presence of a weak space-time segregation phenomenon (downside peaks) within a short spatial distance and a temporal lag of approximately 4 years. These characteristics, in turn, are evidences of a tendency to locate in space and time further away from the existing firms. However, as already done before, these visual considerations need to be supported by a more formal statistical testing procedure. In order to obtain this, we start by inspecting Figure 7 that reports the plot of the estimated standardized residuals of $\hat{R}(d,t)$ against $\hat{K}_D(d)\hat{K}_T(t)$ for the Information technology group.

Figure 7 shows that most of the residuals (99.5% of points) lay within the ± 2 standard deviations. Thus the diagnostic test of residuals shows that the observed tendency to segregation is not substantive. Therefore the space-time segregation phenomenon, observed when commenting on Figure 6, is not substantial and it is only apparent.

**Figure 7**: Plot of the estimated standardized residuals of $\hat{R}(d,t)$ against $\hat{K}_D(d)\hat{K}_T(t)$ for the Information Technology group.

This conclusion is corroborated by the Monte Carlo test for space-time interaction. To run a formal Monte Carlo test of randomness, Figure 8 reports, as before, the sum of the differences between the space-time $K$-function and the product of the separate space and time $K$-functions as they occurred in the 999 simulations. This sum in the observed dataset ranked 576 out of 1000. Therefore, the empirical $p$-value of the test is 0.424, thus providing formal evidence for the space-time randomness in the plot of $\hat{D}_0(d,t)$ and supporting the fact that the weak negative interaction between the spatial and temporal component processes is occurred by chance and is not driven by a systematic underlying phenomenon. As a consequence, even if in the Rome area the ICT industries as a whole tend to be clustered both in space and time, those belonging to the Information technology group present spatial agglomeration in each time period, but no significant interaction between space and time. This behaviour is similar to the stylized fact presented in Figure 1 case v).

Let us now move to comment on similar graphics and test for the Electronic and Communication group. Figure 9 reports the plot of the space-time $K$-functions computed for the 66 Electronic and Communication industries observed in the area of Rome. Again, as in Figure 3, we find evidences of a space-time clustering at short distances with a peak around zero distance and at a temporal interval of 1. The Electronic and Communication firms therefore, display a similar pattern to that observed for the ICT industries considered as a whole.
Figure 10 reports the standardized residuals originated by 999 random permutations of the industrial sites plotted against the product of the separate spatial and temporal $K$-functions. A high share of the residuals (46.5%) lays above the 2 standard deviations line supporting the hypothesis of dependence between the spatial and temporal component processes and the significance of the considerations made on the previous graph.

Finally Figure 11 displays the frequency distribution of the sum of the differences between the space-time $K$-function and the product of the separate space and time $K$-functions in the 999 simulations. The sum of such differences in the observed dataset ranked 993 out of 1000 leading to an empirical $p$-value of 0.007, and hence providing a probabilistic significance to the previously observed space-time clustering pattern.

**Figure 8**: Empirical frequency distribution of the sum of the differences between the space-time $K$-function and the product of the separate space and time $K$-functions in 999 simulations. *Information technology* group.

![Empirical frequency distribution](image1)

Summing up, the *Information technology* and the *Electronic and Communication* groups have a different spatial behaviour. The *Information technology* industries tend to locate in space with no remarkable space-time interaction. Conversely the *Electronic and Communication* companies tend to display a marked agglomeration pattern both in space (at small distances) and time. This dynamic effect is so strong that it is the one that dominates if we look at the *ICT* sector as a whole.

**Figure 9**: 3-dimensional plot of the (a) $\hat{D}(d,t)$ function and (b) $\hat{D}_0(d,t)$ function for the *Electronic and Communication* group.

![3-dimensional plots](image2)
4. Discussion and analysis of the economic implications

The empirical findings reported in Section 3 clearly display a different spatial pattern in the distribution of firms belonging to the Information technology and those belonging to the Electronic and Communication groups within the ICT sector. The observed clustering process for the sector as a whole is mainly due to the very strong agglomeration pattern displayed by the industries belonging to the Electronic and Communication group, while the Information technology companies do not display any significant tendency to space-time interaction in the formation of clusters. As a matter of fact the industries belonging to the ICT sector are quite heterogeneous and they display different managerial and organization behaviour. In fact, the industries belonging to the Information technology group located in our study area are mainly branches of medium-large
multinational companies (the name are not reported here for privacy reasons). These enterprises have in mind a network that is global rather than local. Thus, in their location choices they mainly tend to be present in the big metropolitan areas (basically Rome and Milan in Italy), rather than to distribute evenly in the entire territory. This global network manifests itself mainly in the form of the *global city* described by Sassen (1994), that is more with the aspect of a production process than as a place in the conventional meaning: a process in which geography plays a very limited role and where the production and the consumption centres are interlinked on the basis of information flows and no more on the basis of physical flows between the geographical space. The advent of a new, high-tech, manufacturing industry assisted by computers and microelectronics, led to a new logic in the localization processes. Historically the *Information technology* industries were those that started this new form of spatial location based on information. Such a location pattern is characterised by the technological and managerial ability to split the production process into different places and to integrate them subsequently through computerized links (Castells, 1996). The irregular spatial distribution of activities that is thus produced was already observed empirically by, e.g., Gordon (1994) who also noticed that the new distribution (determined more by information than by geography) produced also, as a by-product, a new spatial division of labour, characterized by *variable geometries* and by reciprocal links between industries that are located within spatial agglomerations (those that are termed *innovation milieu*; see Camagni, 1991; Castells, 1996).

With these premises one may think that in the *ICT* market the only driving force is what Cairncross termed the *death of distance* (see Cairncross, 2001) that is the phenomenon of a space-time compression generated by the possibility of communicating in real time with any point in the world as if everything took place in just one single, dimensionless, point (see also Quah, 1993). However the realization of a production process of this kind requires a direct and physical interaction among entrepreneurs, managers and specialized workers that are in charge of integrating competences that are very different on one another.

This behaviour could be at the basis of the observed space-time clustering pattern in the Rome area for the *Electronic and Communication* industries and for the *ICT* firms as a whole despite the distribution with no space time interaction observed for the *Information technology* group.

In other words the *ICT* industries, rather than totally eliminating the relevance of space in their location decisions, increase the need for a spatial concentration of some activities that contribute to the dispersion of other activities and are in support of the integration among them (Sassen, 1994).

In order to strengthen the idea of global networking that seems to emerge from the previous analysis, we run a further study in which we split the data into two sub-samples: one before and one after internet became widespread. Our theoretical expectations are that if there is less spatial clustering in the latter period, one could prove the claim of spatial location irrelevance.

The first regulation of the networking of IXPs (Internet exchange points) was introduced in Italy around the year 1995. Starting from this consideration we decided to use conventionally this year as a cut-off point. We repeated our analysis separately for the two sub-samples restricting ourselves to only the *ICT* sector as a whole (rather than looking at *Information technology* and *Electronic and Communication* separately) due to the small number of firms in the second sub-period. The results are summarised in Table 2 and seem to confirm our hypotheses. In fact we observe significant spatial (at 1 mile) and temporal (at 1 year) clustering in the first period (before 1995) and conversely no significant clustering both in space and time in the second one (beginning from 1995).
Table 2: Characteristics of the space-time clustering situation of the ICT sector before and after internet widespread.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Before internet widespread (before 1995)</th>
<th>After internet widespread</th>
</tr>
</thead>
<tbody>
<tr>
<td>spatial lag</td>
<td>1 mile</td>
<td>no space-time interaction</td>
</tr>
<tr>
<td>temporal lag</td>
<td>1 year</td>
<td></td>
</tr>
<tr>
<td>p-value Monte Carlo test</td>
<td>0.011</td>
<td>0.592</td>
</tr>
<tr>
<td>% of estimated residuals out of ±2 SE</td>
<td>40.1%</td>
<td>0.50%</td>
</tr>
</tbody>
</table>

5. Conclusions and research priorities

In this paper we have introduced in the economic literature a set of tools, proposed in the spatial statistical literature, to analyse simultaneously the spatial arrangements of firms, their temporal trends and the interactions between the spatial and temporal components of growth. These tools are based on the family of \( K \)-functions and fall within the realm of the so-called marked point pattern analysis. They were introduced, in an epidemiological context, in the seminal work of Diggle et al. (1995) and, to our best knowledge, it is the first time that they are used in the regional sciences.

We argue that these tools may help the economic analysis of spatial clusters of firms by providing a proper and precise way to represent the stylized facts describing the localization processes which are to be explained by theoretical models. Moreover, they allow to empirically verify whether the time dimension is relevant in explaining the spatial arrangements of an industrial situation. Therefore, our work has not the purpose of giving actual consistency to specific spatial economic models. The scientific concern in the present context is to define a way to measure the strength of the tendency for industries to cluster by taking into account the dynamic evolution of the process leading to the observed facts.

In the empirical part of this paper we have shown that the space-time \( K \)-function is an appropriate tool to uncover the process of firm demography, both under a spatial and a temporal point of view thus being able to treat trends and cycles in time and spatial agglomeration phenomena within the same methodological framework. In particular we have applied the proposed methodology to the space-time distribution of the ICT firms in the area of Rome in the time period between 1920 and 2005.

In this respect we obtained the following substantial findings:

- ICT firms considered as a whole tend to display a marked tendency to agglomerate in space and, furthermore, the process of firm creation in time presents a significant space-time interaction. New firms thus tend to be created in the neighbourhood of the existing one.
- A very similar and significant pattern is detected for the subgroup constituted by only the Electronic and Communication industries.
- In contrast, the subgroup constituted by only the Information Technology firms presents a distinctive feature with respect to the whole ICT sector. While presenting a (less marked) tendency to agglomerate in space likewise the Electronic and Communication and ICT industries, they do not display any significant space-time interaction. The process of firm creation in time thus follows a dynamic that is independent from the spatial location of existing firms.
- Splitting the time period into two samples: one before and one after internet became widespread in 1995, we observed significant spatial (at 1 mile) and temporal (at 1 year) clustering in the first period and conversely no significant clustering both in space and time in the second one for the ICT sector as a whole. This result seems to confirm the hypothesis of spatial location irrelevance in the era of internet.
While presenting the theoretical and the empirical results, in this paper we aimed at making it clear the usefulness of the proposed methodology. Once the presence of a specific space-time interaction phenomenon has been detected, the researcher is in the position to model specific hypotheses concerning the geographical and dynamical configurations of economic activities. Under this respect there are many possible questions that can be addressed using the proposed methodology that we plan to tackle in future studies. We review some of them here below while describing the limitations of the examples reported in the present work and, in this way, delineating the future agenda in the field.

A first advance with respect to the work presented here is represented by the extension of the analysis to the inter-type $K$-function approach proposed by Lotwick and Silverman (1982) and applied in an economic context by Arbia et al. (2008). This tool enables us to detect more into depth the complex space-time processes that may occur in practice. For instance, by categorizing the point process in terms of year of establishment, we could test whether phenomena of geographic segregation or aggregation between ‘old’ and ‘young’ firms occur, hence indicating the presence of specific leader-follower patterns. In addition we could test the co-agglomeration dynamics between different sectors.

The space-time $K$-functions used in the present context are built under the basic assumption of stationarity and isotropy of the underlying generating process (Diggle, 2003; Arbia, 2006). In other words, the geography of firms is considered substantially observed on a homogeneous space. This in turn implies that we do not consider the possible presence of physical or administrative limits that could introduce strong constraints in the location choices of firms. As a consequence, one of our future research priorities will consist in removing this assumption that is often violated in an economic context. We argue that possible methods to disentangle spatial heterogeneity and spatial aggregation phenomena could be based on the integration of inhomogeneous $K$-functions (Baddeley et al., 2000) in a space-time perspective. This approach have been followed in Arbia et al. (2009), but neglecting the time dimension. A second possible approach to tackle this problem of heterogeneity could consist in removing the potential heterogeneous sub-areas from the initial study area, thus obtaining a homogenous map. However this solution is not entirely satisfactory because it introduces extra complexity by leading the researcher to analyse irregular polygonal surfaces rather than rectangular areas as it happened for instance in the present context. As a consequence the analysis should also include methods for correcting edge effects when computing space-time $K$-function in study areas of complex shape (see Goreaud and Pélissier, 1999).

A further limitation of the present study consists of the fact that, for a correct analysis of firm demography, we should consider not only the process of birth of new firms, but also the process of growth of the existing ones and the space-time dynamics of the firms that cease their activity in the span of period considered (see e.g. Arbia et al., 2009). Under this respect in the present context we did not take into account the aspects of firm growth and we totally neglected in our analysis the firm dimension (as measured, e.g., by the number of employees or the value added). However, when studying the pattern of industrial agglomeration, the firm dimension is of paramount importance in that a pattern of increased agglomeration of firms can be equally due to a higher number of firms concentrating in the same area or, alternatively, to the firms expanding their dimension. In contrast, in the present context we considered each economic activity in space as a dimensionless point so that what we have detected here was the mere geographic concentration of firms and not the more general concept of industrial agglomeration suggested, e.g., by Duranton and Overman (2005). An important step forward in the analysis of firm clustering in space and time will be constituted by removing this strong limitation and by considering marked point patterns where the marks refer not only to different time periods (as we do in the present context), but also to different firm dimensions. A final point refers to the consideration of the death of the existing firms, an aspect that was also left aside in the present paper. While a correct approach to firm demography should consider jointly the process of firm creation and that of firms ceasing their activity, under the
methodological point of view this adds extra complexity in that the spatial pattern and the interaction between space and time, should be evaluated separately in each time period and not as the resulting process at a given moment of time as we did here when analysing the empirical data on the ICT firm distribution in Rome. Methodological tools should be developed in future researches to overcome this further limitation.

References


Essay 2

Measuring industrial agglomeration with inhomogeneous $K$-function: the case of ICT firms in Milan (Italy)
Measuring industrial agglomeration with inhomogeneous $K$-function:  
the case of ICT firms in Milan (Italy)

**Abstract:** A series of recent papers (Duranton and Overman, 2005; Arbia *et al.,* 2009; Marcon and Puech, 2010) has introduced Ripley’s $K$-function (Ripley, 1977) based explorative methods to analyse the micro-geographic patterns of firms. In particular, the proposed methods are characterized by the ability to detect the spatial dependence between economic activities while controlling for the heterogeneity of the territory where they are located. Although following different approaches, all these papers handle the spatial heterogeneity of the underlying generating process by developing relative measures of spatial concentration, and hence are not directly comparable across different economies or countries. In this paper we suggest a parametric approach based on the inhomogeneous $K$-function (Baddeley *et al.,* 2000) that allows to obtain an absolute measure of agglomeration of economic activities which is still able to capture spatial heterogeneity. In order to show the potential of the proposed methodology, we present an empirical application to study the spatial distribution of ICT firms in Milan (Italy) in 2001.

**Keywords:** Agglomeration, Parametric measures; Inhomogeneous $K$-function, Spatial clusters, Spatial econometrics.

**JEL classification codes:** C21 · D92 · L60 · O18 · R12

1. Introduction

A series of recent papers (Duranton and Overman, 2005; Arbia *et al.,* 2009; Marcon and Puech, 2010) have introduced explorative methods based on Ripley’s $K$-function (Ripley, 1977) to analyse the micro-geographic patterns of firms. In particular, the proposed methods are characterized by the ability to detect the spatial dependence between economic activities while controlling for the heterogeneity of the territory where they are located. Although following different approaches, all these papers handle the spatial heterogeneity of the underlying generating process by referring to a case-control design. In such a setting, spatial clusters manifest themselves as a phenomenon of extra-concentration of one industry with respect to the concentration of the firms in the whole economy. Therefore a positive (or negative) spatial dependence between firms is detected when the pattern of a specific sector is more aggregated (or more dispersed) than the one of the whole economy. As a matter of fact, the tools proposed in the quoted papers are relative measures of the spatial concentration and hence are not straightforwardly comparable across different economies. In this paper we suggest a parametric approach based on the so-called inhomogeneous $K$-function (Baddeley *et al.,* 2000), a tool that produces an absolute measure of the industrial agglomeration which is also able to capture spatial heterogeneity.

In order to show the potential advantages of the proposed method, we present here an empirical application to the study of the spatial distribution of high-tech industries in Milan (Italy) in 2001.

In order to achieve this aim we structured the paper in the following way. In Section 2 we will introduce the basic concepts of spatial heterogeneity and spatial dependence. Section 3 will be devoted to develop the statistical framework and the appropriate parametric model for capturing the
lack of homogeneity of the economic space. In this section we will discuss the inhomogenous $K$-functions and their use in the analysis of the spatial distribution of firms. Section 4 illustrates a simulation study and Section 5 contains an empirical application of the proposed methodology to the study of the spatial distribution of the ICT manufacturing industry in Milan (Italy). We will consider the economic space to be non homogenous, we will estimate the pattern of inhomogeneity and we will use it to separate spatial heterogeneity form spatial dependence. Finally Section 6 reports some general conclusions.

2. Phenomena behind spatial concentration of firms: spatial heterogeneity and spatial dependence

In a micro-geographical context the spatial distribution of economic activities is properly represented by a set of points in a planar map (formally a point pattern) where each point corresponds to the location of a single firm.

A point pattern which exhibits spatial clusters of events can be originated by two distinct phenomena which can be traced back to the statistical categories of true contagion and apparent contagion between points (Arbia and Espa, 1996).

Applying these concepts to industrial agglomeration problems, the case of apparent contagion arises if exogenous factors lead firms to locate in certain specific geographical zones. For instance, firms may group together in certain areas in order to exploit favourable local conditions, such as the presence of useful infrastructures, the proximity to the communication routes or more convenient local taxation systems.

The case of true contagion, on the other hand, occurs when the presence of an economic activity in a given area attracts other firms to locate nearby. For instance, the presence of firms with a leading role encouraging the settlement of firms producing intermediate goods in the same area or the incidence of knowledge spillovers driving industrial agglomerations.

In more formal terms, spatial heterogeneity (produced by an apparent contagion), is represented through the first-order intensity of a spatial point pattern which in turn is expressed by the so-called intensity function, say $\lambda(x)$, with $x$ representing the geographic coordinates of an arbitrary point. Intuitively, $\lambda(x)dx$ expresses the probability that an event locates inside an infinitesimal region centred at point $x$ and with a surface area $dx$ (Diggle et al., 2007). If a point process (which is a stochastic mechanism generating points in a pattern) is homogeneous, then the intensity does not vary across the space and $\lambda(x) = \lambda$ for each $x$. The constant $\lambda$ can then be interpreted as the expected number of events occurring within a unitary region (Diggle, 2003).

In contrast, the case of spatial dependence (produced by a true rather than apparent contagion) can be expressed by the second-order intensity $\lambda_2(x,y)$, where $x$ and $y$ denote the geographic coordinates of two distinct arbitrary points. We can also define $\lambda_2(x,y)dx dy$ as the probability that two events locate inside two infinitesimal regions centred in $x$ and $y$ and with surface areas $dx$ and $dy$ respectively (Diggle et al., 2007). Therefore, $\lambda_2(x,y)$ characterizes the expected additional events located in $y$ relative to a given event located in $x$. Hence it represents a measure of spatial dependence.

In real cases clusters of firms could be generated by the joint action of spatial heterogeneity and spatial dependence, with the second phenomenon often of paramount interest in economics. Under a substantive point of view, if it is important to measure properly the concentration of economic activities, it is fundamental to clearly distinguish between spatial heterogeneity and spatial dependence. In other words, a statistical tool that aims at detecting the genuine attraction between economic activities (what we call spatial dependence) should be able to control for the different opportunities offered by the territory where firms are located which is what we refer to as heterogeneity. This aim is accomplished in the next section.
3. The statistical methodological framework

3.1 Inhomogeneous K-function

It is probably not an exaggeration to affirm that Ripley’s K-function (Ripley, 1976 and 1977) is currently the most popular measure to summarize the spatial distribution of micro-geographic data where space is assumed to be homogeneous. Such an approach has been largely applied in various fields like, e.g., geography, ecology, epidemiology and, more recently, also economics (see Arbia and Esco, 1996; Marcon and Puech, 2003). The tool of the K-function can be conceived as a measure of the second-order intensity, say \( \lambda_2(x,y) \), of the underlying stationary point process which generates the observed spatial pattern. It is a function (usually denoted with the symbol \( K(d) \)) that, at every spatial distance \( d \), reports the expected number of additional points located in a circle of radius \( d \) surrounding an arbitrary event. As a consequence, in the case of geographic homogeneity of the study area (when \( \lambda(x) = \lambda \)), the K-function quantifies the spatial dependence between events at each unit of distance.

However, if \( \lambda(x) \) varies across the space, then the values of the K-function express the magnitude of the spatial concentration pattern due to both dependence between events and geographic heterogeneity jointly without being able to distinguish on an empirical basis the two phenomena characterizing the spatial concentration of firms: interactions among economic agents and exogenous features of the territory. Baddeley et al. (2000) introduced in the spatial statistical literature an instrument that enables this distinction. It is a non homogeneous version of Ripley’s K-function which can be used to assess the endogenous effects of interaction among events while adjusting for the exogenous effects of the characteristics of the study area. In an industrial agglomeration context this tool can be properly employed to test for the presence of genuine spatial interactions among economic activities discounted of the effect of a heterogeneous geographic space.

The inhomogeneous K-function, say \( K_I(d) \), is essentially a generalization of Ripley’s function to the case of non-stationary point processes in which second-order intensity-reweighted stationarity is assumed (Baddeley et al. 2000). More precisely, a non-stationary point process is a homogeneous Poisson process where the constant intensity \( \lambda \) is replaced by an intensity function varying over the space, say \( \lambda(x) \). By considering a hypothetical study region \( A \) with a surface area \( |A| \), the class of inhomogeneous Poisson processes is characterized by the following two postulates (Diggle, 2003):

(i) the \( n \) points located in \( A \) follow a Poisson distribution with expected value given by

\[
\int_A \lambda(x) dx ; \quad \text{and}
\]

(ii) the \( n \) points located in \( A \) constitute an independent random sample from the distribution on \( A \) with probability density function proportional to \( \lambda(x) \).

Following Baddeley et al. (2000), an inhomogeneous Poisson process is also second-order intensity-reweighted stationary if

\[
\lambda_2(x,y) = \lambda(x)\lambda(y)g(x-y)
\]

where \( g(x-y) \) is a function which depends on the spatial lag (and hence on the interaction) between the \( x \) and \( y \) arbitrary events. In this case, the second order intensity of the inhomogeneous
Poisson process, at the geographic locations \( x \) and \( y \), is the product of the first order intensities at \( x \) and \( y \) multiplied by a spatial correlation factor. If there is no spatial interaction between the points of the process at locations \( x \) and \( y \) then \( \lambda_2(x, y) = \lambda(x)\lambda(y) \) and \( g(x - y) = 1 \). Furthermore, when \( g(x - y) > 1 \) we have attraction, while if \( g(x - y) < 1 \) we have repulsion (or inhibition) between the two locations (Møller and Waagepetersen, 2007).

In the case of isotropy (when no directional bias occurs in the neighbourhood of each point, see Arbia, 2006), \( g() \) depends only on the distance between \( x \) and \( y \), thus implying that \( g(x - y) = g\|x - y\| \), where \( \| \) is the Euclidean norm. In spatial statistics literature the term \( g\|x - y\|) = \lambda_2(x, y) / \lambda(x)\lambda(y) \) is referred to as the pair correlation function.

If \( \lambda(x) \) is bounded away from zero, the \( K \)-function of a second-order intensity-reweighted stationary and isotropic spatial point process is given by (Baddeley et al., 2000; Diggle, 2003):

\[
K_i(d) = 2\pi \int_0^d \mu(g(u)du, \text{ with } d > 0)
\]  

(1)

For an inhomogeneous Poisson process without spatial interactions between events, we have \( K_i(d) = \pi d^2 \). On the other hand, when \( K_i(d) > \pi d^2 \) (or \( K_i(d) < \pi d^2 \)) the point pattern is more (or less) aggregated than a point pattern drawn from an inhomogeneous Poisson process with first-order intensity \( \lambda(x) \) and no spatial interactions (Diggle et al., 2007).

Following Baddeley et al. (2000), if \( \lambda(x) \) is known, a proper edge-corrected unbiased estimator of \( K_i(d) \) is

\[
\hat{K}_i(d; \lambda) = \frac{1}{|A|} \sum_{i=1}^{n} \sum_{\substack{j \neq i}} w_{ij} I\{d_{ij} \leq d\} / \lambda(x_i)\lambda(x_j)
\]  

(2)

where \( |A| \) is the total surface of the study area, the term \( d_{ij} \) is the Euclidean spatial distance between the \( i \)th and \( j \)th observed points and \( I\{d_{ij} \leq d\} \) represents the indicator function such that \( I = 1 \) if \( d_{ij} \leq d \) and 0 otherwise. Due to the presence of edge effects arising from the arbitrariness of the boundaries of the study area, the adjustment factor \( w_{ij} \) is introduced thus avoiding potential biases in the estimates close to the boundary.\(^2\)

The functional form of the first-order intensity \( \lambda(x) \) is unknown and it has to be estimated from the data. If one wishes to estimate it non-parametrically, the suitable procedure is the well known kernel smoothing (see Silverman, 1986). Baddeley et al. (2000) suggest to use the following kernel estimator

\[
\hat{\lambda}_h(x_i) = \sum_{j \neq i} \frac{h^{-2} k\left(\frac{x_{ij} - x_i}{h}\right)}{C_h(x_j)},
\]  

(3)

\(^2\) More specifically, the weight function \( w_{ij} \) expresses the reciprocal of the proportion of the surface area of a circle centred on the \( i \)th point, passing through the \( j \)th point, which lies within \( A \) (Boots and Getis, 1988).
where \( k(\cdot) \) is a radially symmetric bivariate probability density function, \( h \) represents the bandwidth (the parameter controlling the smoothness of the surface), and \( C_h(x_i) \) is an edge-correction factor\(^3\).

Baddeley et al. (2000) argue that, with a proper choice for the bandwidth \( h \), the estimator of the inhomogeneous \( K \)-function, \( \hat{K}_J(d) \), which incorporates Equation (3) for the estimation of the first-order intensity, is approximately unbiased. However, as pointed out by Diggle et al. (2007), if we aim to estimate non-parametrically both \( \lambda(x) \) (the first-order structure) and \( K_J(d) \) (the second-order structure), using the same observed point pattern, a practical problem arises. Indeed, without any other information or explicit assumption about the nature of the underlying point process, we are not able to distinguish the contributions due to spatial heterogeneity or spatial dependence phenomena and, as a result, we risk to have spurious estimates.

To tackle this problem, depending on the field of application, it may be plausible to assume that the spatial scale of the first-order intensity is larger than the spatial scale of the second-order intensity (Diggle et al., 2007). In other words, the heterogeneity of the environment operates at a larger scale than the one characterizing spatial interactions among events, and, as a consequence, these two characteristics of the underlying point process are separable. This specific assumption, however, may be too narrow or not realistic enough.

In a different way, following Diggle (2003), we suggest that \( \lambda(x) \) can be properly estimated by the means of a parametric regression model where \( \lambda(x) \) is specified as a function of a set of geographically-referenced variables expressing spatial heterogeneity (e.g., proximity to main roads, presence of infrastructure, presence of public incentives and so on). A plausible specification of the model for \( \lambda(x) \) to describe how the probability of hosting a firm changes through space is the log linear model:

\[
\lambda(x) = \exp \left\{ \sum_{j=1}^{m} \beta_j z_j(x) \right\}
\]  

(4)

where \( z_j(x) \) is the \( j \)th of a set of \( m \) spatially referenced explanatory variables and the \( \beta_j \)'s are the regression parameters.

The logarithmic transformation allows to fit the model by maximizing the log-pseudolikelihood (see Besag, 1975) for \( \lambda(x) \) based on the observed points \( x_i \) of the pattern under study. At the current state of the spatial statistics literature, the most efficient and versatile method to maximize the log-pseudolikelihood and then to obtain the estimates of \( \beta_j \)'s parameters and, as a result, \( \lambda(x) \) is that proposed by Berman and Turner (1992). For a clear and detailed discussion of the method we refer to Baddeley and Turner (2000).

As shown by Strauss and Ikeda (1990), in case of a Poisson stochastic process, maximum pseudolikelihood is equivalent to maximum likelihood. Therefore, it is possible to test for goodness of fit of the model expressed in Equation (4) by using standard formal likelihood ratio criteria and the \( \chi^2 \) distribution.

The estimation of \( \lambda \) thus obtained (say \( \hat{\lambda} \)) can be used to replace the true value of \( \lambda \) in Equation (2) in order to obtain the estimated function \( \hat{K}_J(d) \). For the ease of interpretation, similarly to the case of Ripley’s homogeneous \( K \)-function, also in the case of the inhomogeneous function we can introduce the linearizing transformation proposed by Besag (1977) which is characterized by a more

\[^3\] The kind of edge-correction factor Baddeley et al. (2000) used is essentially a slightly modified version of that proposed in Berman and Diggle (1989), \( C_h(x_i) = \int k_h(x_i - u) du \).
stable variance. In the non-stationary case such a transformation function assumes the following expression:

\[ \hat{L}_I(d) = \sqrt{\hat{K}_I(d)} / \pi \]  

(5)

where the function is linearized dividing by \( \pi \) and to stabilized the variance by the square root. If we use such a normalization, under the null hypothesis of absence of spatial dependence we have \( \hat{L}_I(d) = d \).

3.2 Inference

In order to evaluate the statistical significance of the values of \( \hat{K}_I(d) \) (or \( \hat{L}_I(d) \)), which measure the strength of the spatial interactions among events within an inhomogeneous space, a proper inferential framework needs to be introduced. Since the exact distribution of \( K_I \) is unknown, its variance cannot be evaluated theoretically and no exact statistical testing procedure can be adopted. Therefore, to construct confidence intervals for the null hypothesis of absence of spatial interactions, we will base our conclusions on Monte Carlo simulations (Besag and Diggle, 1977).

In practice, we generate \( n \) simulations of inhomogeneous Poisson processes conditional upon the same number of points of the observed pattern and with probability density function proportional to the first-order intensity, \( \lambda(x) \), estimated in the study area. Then for each simulation we can calculate a different \( \hat{L}_I(d) \) function. We are then able to obtain the approximate \( n/(n+1) \times 100\% \) confidence envelopes from the highest and lowest values of the \( \hat{L}_I(d) \) functions calculated from the \( n \) simulations under the null hypothesis. Finally, if the observed \( \hat{L}_I(d) \) falls, for some value of \( d \), outside the envelopes – upward or downward – this will indicate a significant departure from the null hypothesis.

In the next sections, in order to simulate the conditioned inhomogeneous Poisson processes to analyze inferentially the empirical results, we will follow the ‘thinning’ computational algorithm suggested by Lewis and Shedler (1979). First of all, we generate \( n \) points following a homogeneous Poisson process on the study region and then we delete each point, independently of other points, with deletion probability \( \hat{\lambda}(x)/\lambda_0 \), where \( \lambda_0 \) is equal to the maximum value of the estimated first-order intensity function \( \hat{\lambda}(x) \).

4. A simulation study

To assess the performance of the proposed method, we generated two paradigmatic artificial point patterns of economic activities. Both are characterized by the presence of spatial heterogeneity, which drives economic agents to locate in specific sub-areas, but one pattern exhibits spatial interaction between agents while the other does not show any evidence of spatial dependence. In this simulation design example, spatial heterogeneity arises from some stochastically generated spatially referenced covariates.

In order to run simulations proper to represent realistic cases, we considered covariates representing spatial heterogeneity (such as locations of access to the main communication routes, locations of industrial infrastructures, institutions, services for firms, and so on) to have a similar spatial behaviour and hence to be spatially correlated. Indeed, in reality we may expect, for instance, that
both accesses to the communication routes and industrial infrastructures are more present in the big urban centres than in the small rural areas. To introduce this spatial similarity we generated three covariates \( z_1, z_2 \) and \( z_3 \) – by referring to the same underlying spatial intensity. Specifically, all covariates are point patterns which are partial realizations of the same Gaussian Cox process, that is an inhomogeneous Poisson process where the varying spatial intensity, \( \lambda(x) \), is in turn a realization of a Gaussian random field, \( \{\Lambda(x)\} \) (see Møller et al., 1998; Diggle, 2003).

Therefore, we first simulated the common underlying intensity surface, \( \{\Lambda(x) = \lambda(x) : x \in \mathbb{R}^2\} \), on the unit square for the three covariates, where \( \Lambda(x) \) is a Gaussian random field with mean \( \mu = 100 \), variance \( \sigma^2 = 0.25 \) and correlation function \( \rho(d) = \exp\{-d/0.25\} \). By way of illustration, Figure 1 shows the simulated underlying intensity surface for the three covariates in which lighter are the grey colours higher is the intensity.

**Figure 1:** Simulated underlying intensity surface for the three covariates \((z_1, z_2 \text{ and } z_3)\) on the unit square (grey-scale image)

Then, conditional on \( \{\Lambda(x) = \lambda(x) : x \in \mathbb{R}^2\} \), we generated the point patterns for the three covariates from an inhomogeneous Poisson process with the intensity function \( \lambda(x) \).

Since, in order to fit the regression model (3), we need the covariates to be measured continuously through the study area (see Berman and Turner, 1992), we converted the three point patterns as continuous surfaces: \( z_1(x) \), \( z_2(x) \) and \( z_3(x) \). In particular, for each point pattern we estimated its intensity function using the method of kernel smoothing (see Silverman, 1986). The kernel estimator of the intensity of a point pattern takes the following form (Diggle, 1985; Diggle, 2003):

\[
\hat{\lambda}_h(x) = \frac{\sum_{i=1}^{n} h^{-2} k\left(\frac{x - x_i}{h}\right)}{C_h(x)},
\]

where \( k \) is a radially symmetric probability density function, the bandwidth \( h \) is a positive real number and \( C_h(x) = \int_{A} k_h(x-u) du \), with \( A \) representing the study area, is a factor proposed by Berman and Diggle (1989) to correct for the presence of edge effects. For all covariates we opted for a Gaussian probability density function and a bandwidth \( h = 0.05 \). Figure 2 shows the graphs of the three covariates both in form of point pattern and intensity surface.
Figure 2: Graphs of the three simulated covariates, as point patterns and intensity surfaces (grey-scale images)

$z_1$:

$z_2$:

$z_3$:
In order to simulate the two paradigmatic point patterns of economic activities within a space which is heterogeneous because of the influence of the three covariates, we considered the joint location of the three covariates as the source of spatial heterogeneity. In particular, the two paradigmatic patterns are both realizations of inhomogeneous processes where the intensity function, that we denote with $\lambda_c(x)$, has been estimated upon the point pattern formed by all the locations of the covariates jointly considered. For the estimation we used again the kernel smoothing technique described above.

The hypothetic pattern of economic activities characterized by absence of spatial interactions among agents (from now on labelled as pattern A) has then been simply generated from an inhomogeneous Poisson process with intensity function $\lambda_c(x)$ on the unit area. On the other hand, the hypothetic pattern characterized by positive spatial dependence, and hence clustering, (from now on labelled as pattern B) has been simulated following an inhomogeneous version of the Poisson cluster process. That is, firstly some agents, say the leaders, have been allocated on the unit area according to $\lambda_c(x)$; then other agents, say the followers, has been allocated in the proximity of each leader on the basis of a radially symmetric Gaussian distribution centred on the leader’s position. Specifically, the probability density function of this distribution, which generated the locations of the followers respect to each leader, is the following:

$$h(x_1, x_2) = \left(2\pi\sigma^2\right)^{-1} \exp\left[-\left(x_1^2 + x_2^2\right)/2\sigma^2\right]$$

where $(x_1, x_2)$ are the geographic coordinates of a follower, and $\sigma$, that we set to be equal to 0.03, is a parameter representing the maximum spatial extension of the area of influence for each leader.

Figure 3 shows the graphs of pattern A and pattern B. The simulations have been run using the same random number seed, and conditioning to the same number of points, so that the differences between the two patterns can only been ascribed to the absence or presence of spatial interactions.

Figure 3: Graph of the two paradigmatic simulated inhomogeneous point patterns of economic activities: without spatial interactions among agents (pattern A), with spatial interactions among agents (pattern B)

We then use pattern A and pattern B to illustrate the application of the method, described in Section 3, to detect spatial clustering while controlling for the presence of spatial heterogeneity. For both patterns we estimate the spatial intensity by using the model expressed in Equation (4), with the following specifications:
Mod0: \( \lambda(x) = \exp(\alpha) \)

Mod1: \( \lambda(x) = \exp\{\alpha + \beta_1 z_1(x) + \beta_2 z_2(x) + \beta_3 z_3(x)\} \)

Mod2: \( \lambda(x) = \exp\{\alpha + \beta_1 z_1(x) + \beta_2 z_2(x) + \beta_3 z_3(x) + \beta_4 z_4(x)\} \)

The first specification, which identifies an homogeneous Poisson process, represents the null model of absence of spatial heterogeneity. Mod1 assumes that spatial intensity is a function of the explanatory variables \( z_1(x), z_2(x) \) and \( z_3(x) \). Mod2 takes also into account the effects of another explanatory variable, \( z_4(x) \), which is the intensity surface of a point pattern generated from an homogeneous Poisson process unrelated to \( \lambda(x) \), and hence it does not have any effect on spatial heterogeneity. This specification has been considered in order to verify if the inferential procedure allows to recognize whether the effect of an explanatory variable is really significant or not.

All models have been estimated using the method of Berman and Turner (1992) which is implemented in the Spatstat package of statistical software R.\(^4\) The model selection, in terms of parsimony and precision of the estimates, has been conducted through likelihood ratio tests; the results are summarised in Table 1 for pattern A and Table 2 for pattern B. The \( p \)-values (denoted with \( P(>|\chi^2|) \)) in Table 1 and 2) for the likelihood ratio tests between Mod1 and Mod0 show, for both paradigmatic patterns of economic activities, that there is significant spatial heterogeneity due to the effect of variables \( z_1(x), z_2(x) \) and \( z_3(x) \). On the other hand, the result of the test between Mod2 and Mod1 confirms that introducing \( z_4(x) \) does not improve the fit of the model; therefore, such variable is not relevant in affecting spatial heterogeneity.

<table>
<thead>
<tr>
<th>Model</th>
<th>Maximum log-likelihood</th>
<th>log LR test with Mod0</th>
<th>log LR test with Mod1</th>
<th>log LR test with Mod2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mod0</td>
<td>532.121</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Mod1</td>
<td>555.369</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Mod2</td>
<td>555.510</td>
<td>0.000</td>
<td>0.60</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>Maximum log-likelihood</th>
<th>log LR test with Mod0</th>
<th>log LR test with Mod1</th>
<th>log LR test with Mod2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mod0</td>
<td>532.121</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Mod1</td>
<td>540.412</td>
<td>0.001</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Mod2</td>
<td>540.708</td>
<td>0.002</td>
<td>0.44</td>
<td>-</td>
</tr>
</tbody>
</table>

These results lead us to use Mod1 to estimate \( \lambda(x) \) for pattern A and B and, in turn, to estimate \( \hat{L}_i(d) \) using Equations (2) and (5). To formally assess the confidence of the results thus obtained, we derive approximate 99.9% confidence envelopes from 999 simulated realisations of a conditioned inhomogeneous Poisson process. More specifically, at every step of the simulation sequence, an inhomogeneous Poisson process with the same number of points as pattern A (and

---

pattern B) is generated and used to estimate \( \hat{L}_I(d) \). Repeating this step 999 times and taking, for each spatial distance \( d \), the maximum and minimum values of the resulting sequence of \( \hat{L}_I(d) \) obtained, we are able to build up the confidence envelopes for the null hypothesis of no spatial interactions (or dependence) within an inhomogeneous space.

The graphs in Figure 4 displays the behaviour of the functionals \( \hat{L}_I(d) \) for pattern A and pattern B at the various spatial distances \( d \). The same figure also reports the confidence envelopes referred to the null hypothesis of absence of spatial dependence at a significance level \( \alpha = 0.01 \). In the graphs, points outside the envelop highlight significant concentration if they lay above the envelop upper limits or significant dispersion if they are observed below. Coherently with the criteria with which the two paradigmatic patterns have been simulated, \( \hat{L}_I(d) \) for pattern A lays within the confidence envelopes indicating absence of spatial interactions among agents, while \( \hat{L}_I(d) \) for pattern B deviates upward indicating, on the contrary, presence of spatial interactions among agents.

**Figure 4:** Behaviour of the estimated inhomogeneous L-function (solid line) and of the corresponding 99.9% confidence envelopes under the null hypothesis (dashed lines) for pattern A (left) and for pattern B (right).

5. A case study: the distribution of ICT manufacturing firms in Milan (Italy)

For our empirical study we focus on a set of micro data related to the ICT\(^5\) manufacturing industry in the area of Milan (Italy). A dataset was collected in the year 2001 by the Italian National Institute of Statistics (ISTAT) reporting the full address and the number of employees of the 856 manufacturing plants operating in this area. The sector is dominated by small firms in that approximately 85% of ICT firm located in Milan have less than ten employees.

The map reported in Figure 5 displays the spatial distribution of the 856 manufacturing plants. It is clear from a first visual inspection of the graph the marked tendency of the firms to cluster and to concentrate in the south part of the city.

\(^{5}\) In this particular study we consider as ICT firms the manufacturing plants which belong to the ATECO classification codes “Manufacture of office machinery and computers” and “Manufacture of radio, television and communication equipment and apparatus”.

75
Due to the current scarcity of geo-referenced data about spatial heterogeneity, our dataset, as said, contains only the geographic coordinates and the number of employees of the 856 manufacturing plants. As a consequence, for the time being, in our analysis we can only estimate $\lambda(x)$ referring to the spatial coordinates of firms as an element of heterogeneity by assuming a spatial trend. More specifically, in order to estimate the spatial intensity of the process we considered the three following specifications of the model expressed in Equation (3):

Constant trend model: $\lambda(x) = \exp\{\alpha\}$

Linear trend model: $\lambda(x) = \exp\{\alpha + \beta_1 x_1 + \beta_2 x_2\}$

Quadratic trend model: $\lambda(x) = \exp\{\alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2\}$

where $x_1$ and $x_2$ are the Cartesian coordinates of location $x$.

The constant model, which is actually a stationary Poisson process, represents the null model of absence of spatial heterogeneity. The other two specifications represent different ways to model the spatial intensity producing spatial heterogeneity.

The model selection results are summarised in Table 3. The $p$-values (denoted with $p(>\chi^2)$ in Table 3) for the likelihood ratio tests with the constant trend model show that both the linear trend model and quadratic trend model are significant, thus implying that there is significant spatial heterogeneity in the pattern of firms under study. Moreover, the result of the test between the two models with spatial trend indicates that the quadratic trend model has a better fit. As a consequence, we opted for the quadratic spatial trend specification, rather than the simpler linear, because it better captures the main features of the observed variation in the spatial intensity.
Table 3: Model selection: results for the log-likelihood ratio tests

<table>
<thead>
<tr>
<th>Model</th>
<th>Maximum log-likelihood</th>
<th>log LR test with Constant trend model</th>
<th>log LR test with Linear trend model</th>
<th>log LR test with Quadratic trend model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>P(&gt;</td>
<td>$</td>
<td>\chi^2</td>
</tr>
<tr>
<td>Constant trend</td>
<td>7998.531</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Linear trend</td>
<td>8104.360</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Quadratic trend</td>
<td>8692.956</td>
<td>0.000</td>
<td>0.000</td>
<td>-</td>
</tr>
</tbody>
</table>

Higher-order specifications may have a better fit but, on the other hand, they increase the risk of introducing artificial patterns, as it is argued in the literature on polynomial regression models (see, e. g., Magee, 1988). The values of the estimated spatial intensity, \( \hat{\lambda}(x) \), are then given by the following estimated equation:

\[
\hat{\lambda}(x) = \exp\left\{ -2040780.1562 + 1532.2127x_1 + 89444.6378x_2 - 421.4319x_1^2 - 997.2483x_2^2 + 136.6161x_1x_2 \right\}.
\]

Having derived an estimate of \( \lambda(x) \) we can then estimate \( \hat{L}_I(d) \) using Equations (2) and (5). The graph in Figure 6 displays the behaviour of the functional \( \hat{L}_I(d) \) at the various spatial distances \( d \) for the ICT manufacturing industry. The same figure also reports the confidence envelopes referred to the null hypothesis of absence of spatial dependence at a significance level. Observing the graph it is evident a significant upward deviation, respect to the confidence envelopes, of the estimated function at very small distances (below 0.4 kilometers). This suggests that ICT firms in Milan are characterized by a very small-scale aggregation phenomenon.

Figure 6: Behaviour of the estimated inhomogeneous L-function (solid line) and of the corresponding 99.9% confidence envelopes under the null hypothesis (dashed lines) of the ICT manufacturing plants.
The empirical tool used help us to disentangle empirically spatial heterogeneity from the spatial dependence observed in the pattern of the ICT manufacturing industry in Milan. The $L$-function reported in Figure 6 shows clearly how strong is the tendency for plants to cluster because of a genuine interaction among economic agents. Indeed, the $L$-function identifies situations of overconcentration with respect to a non constant underlying intensity explained in terms of a quadratic trend. In this way it clearly quantifies the amount of spatial concentration which cannot be simply ascribed to exogenous factors which change smoothly over space like, e. g., traffic accessibility, factor endowments, law and environmental limits. These factors (here approximated by a spatial trend) could be better identified through a set of explanatory variables, as we did in the simulation study, if these were available.

Despite this lack of information, we can argue that the economic theories on industrial agglomerations may have an important role here in explaining the ICT industrial distribution of the Milan area. Our empirical findings may have a justification in the theoretical models explaining the industrial agglomeration as a phenomenon primarily driven by the presence of spatial interactions between economic activities. The expectation of clustering of ICT firms within the big metropolitan areas, such as Milan, can be ascribed to the so-called tacit knowledge phenomenon (Nonaka and Takeuchi, 1995; Polanyi, 1966) which assumes that the knowledge is transferable only through direct face-to-face interaction (Storper and Venables, 2003). Knowledge spillovers should indeed be more easily present in cities, where many specialized workers are concentrated into a relatively small and limited space and where the transmission of new knowledge tends to occur more efficiently by direct human interaction (Glaeser et al. 1992; Henderson et al. 1995; Dumais et al. 2002; Van Oort 2004; Lambooy and Van Oort, 2005). The role of geographical and cultural proximity for tacit knowledge exchange has been discussed extensively in the literature on high-technology clusters (Saxenian, 1994; Storper, 1997, 2002; Porter, 1998; Keeble and Wilkinson, 2000; Yeung et al. 2007) and innovative milieux (Capello, 1999; Rallet and Torre, 1999) and represent the basis to explain our findings.

6. Conclusions

One of the more controversial issues in the geographical analysis of plant location is the possibility of disentangling spatial dependence and spatial heterogeneity or, in other words, to be able to discern whether firms tend to locate close to one another because they need for physical interaction (spatial dependence or true contagion) or because of the varying opportunity offered by different locations that made more efficient to locate in some areas with respect to others (heterogeneity or apparent contagion). In the analysis presented here we employed a statistical model to estimate separately spatial heterogeneity allowing to test for the presence of absolute spatial aggregation while controlling for the effect of exogenous factors that can be at the basis of diverse location opportunities.

The empirical analysis revealed a strong tendency for plants to cluster at very small distances because of a genuine interaction between them, which cannot be explained simply by exogenous factors such as accessibility, endowments and other institutional elements.

In this paper we limited ourselves to prove this conclusion on empirical basis without trying to validate any theoretical behavioural model. Indeed, in order to propose a theoretical model to properly explain the observed situation, it would be necessary to avail a larger information set on structural variables other than just the mere geographic location, such as, for instance, the characteristics of the local demand and the workforce skill. For this reason this purpose is not undertaken in the present study and is left to future refinements.

However, in the present work we were able to perform the important task of verifying empirically the presence of an endogenous location phenomenon leading to spatial clusters. Furthermore, the
statistical testing procedure we applied is based on an absolute measure of spatial dependence. The advantage of this approach is that the results obtained here can be straightforwardly compared with other geographic areas and industrial situations.

References


Essay 3

Weighting Ripley’s $K$-function to account for the firm dimension in the analysis of spatial concentration
Weighting Ripley’s $K$-function to account for the firm dimension in the analysis of spatial concentration

Abstract: The spatial concentration of firms has long been a central issue in economics both under the theoretical and the applied point of view due mainly to the important policy implications. A popular approach to its measurement, which does not suffer from the problem of the arbitrariness of the regional boundaries, makes use of micro data and looks at the firms as if they were dimensionless points distributed in the economic space. However in practical circumstances the points (firms) observed in the economic space are far from being dimensionless and are conversely characterized by different dimension in terms of the number of employees, the product, the capital and so on. In the literature, the works that originally introduce such an approach (e.g. Arbia and espa, 1996; Marcon and Puech, 2003) disregard the aspect of the different firm dimension and ignore the fact that a high degree of spatial concentration may result from both the case of many small points clustering in definite portions of space and from only few large points clustering together (e.g. few large firms). We refer to this phenomena as to clustering of firms and clustering of economic activities. The present paper aims at tackling this problem by adapting the popular $K$-function (Ripley, 1977) to account for the point dimension using the framework of marked point process theory (Pentinen, 2006).

Keywords: Agglomeration, Marked point processes; Spatial clusters, Spatial econometrics.

JEL classification codes: C21 · D92 · L60 · O18 · R12

1. Introduction

Spatial economics theories show that economic integration may boost spatial concentration of economic activities and industrial specialization both at a regional and at an international level (Bickenbach and Bode, 2008). Furthermore, due to the external increasing returns driven by the spatial concentration, the core regions (where spatial clusters of firms are more likely to occur) may reach higher levels of economic growth than the peripheral regions (see Krugman, 1991 and Fujita et al., 1999 among others). As a consequence, the phenomenon of spatial concentration is of paramount importance to explain the determinants of growth and development on one hand and regional disparities on the other.

Fostered by the centrality of these issues under the theoretical and the practical point of view, a variety of empirical studies have tried to develop proper indices and statistical tests to measure the degree of spatial clustering in real industrial situations. Under this respect, a series of recent papers (Arbia et al, 2008, 2009; Marcon and Puech, 2003, 2010; Duranton and Overman, 2005, 2008) have introduced the use of distance-based methods. These methods are more robust than the traditional measures of spatial concentration (such as Gini index (Gini 1912, 1921) or Ellison-Glaeser index (Ellison and Glaeser, 1997)), which make use of regional aggregates and thus depend on the arbitrariness of the definitions of the spatial units. The distance-based methods, conversely, make use of micro economic data, treating each firm as a point on a map and studying their spatial distribution with the methods borrowed from the so called point pattern analysis (Diggle, 2003).
In many empirical circumstances where the presence of spatial clusters of firms is tested by using micro-geographical data, an important element to be taken into account is represented by the firm dimension. Indeed a high level of spatial concentration can be due to two very different phenomena (see Figure 1). Namely,

- **Case 1**: many small firms clustering in space, and
- **Case 2**: few large firms (in the limit just one firm) clustering in space.

We can refer to the first case as to the case of *clustering of firms* and to the second as to the case of *clustering of economic activities*.

**Figure 1**: Two extreme paradigmatic situations of spatial concentration

A proper test for the presence of spatial clusters should thus consider the impact of the firm dimension on industrial agglomeration by clearly distinguishing these two cases. Under this respect, Marcon and Puech (2010) and Duranton and Overman (2005) have extended the use of Ripley’s *K*-function (Ripley, 1977) considering firm size and treating it as a weight attached to each of the points constituting the pattern. Both quoted papers developed *relative* measures of the spatial concentration, detecting the extra-concentrations of firms belonging to a specific industry with respect to the distribution of firms of the whole economy. Following this procedure a positive (or negative) spatial dependence between firms is detected when the pattern of a specific sector is more aggregated (or more dispersed) than the pattern of the whole economy. Although measures of relative spatial concentration are very useful in controlling for the idiosyncratic characteristics of the territories under study, on the other hand they do not allow comparisons across different economies (see Haaland *et al.*, 1999 and Mori *et al.*, 2005 for a more detailed discussion).

In this paper we propose a similar extension of Ripley’s *K*-function which leads to an *absolute* (rather than a *relative*) measure of the industrial agglomeration and which allows comparability amongst different empirical situations. More specifically, referring to the theory of *marked point processes*, we develop a stochastic mechanism which generates weighted point patterns of firms representing stylized facts of the different phenomena occurring in real cases (essentially: spatial randomness or spatial concentration in the sense indicated in “Case 1” or “Case 2” above).
values assumed by the proposed measure in the various cases constitute the benchmark that allows us to formally test the departure from spatial randomness. We will present our new approach along the following lines. In Section 2 we will briefly discuss the classical Ripley’s K-function which represents the starting point to develop more sophisticated measures of spatial concentration. Section 3 will be devoted to introduce the stochastic mechanism based on the marked point processes theory which allows us to develop a test for the presence of absolute spatial concentration of firms and economic activities. In this section we will introduce the new model, we will discuss the meaning of the model’s parameters in the context of spatial concentration of firms and economic activities and we will present some simulation results to better illustrate how the model works in practice. Finally, Section 4 contains a discussion of the results, some conclusions and directions for further studies in the field.

2. Measuring the spatial concentration of firms disregarding size: the basic K-function

It is probably fair to say that Ripley’s K-function (Ripley, 1976 and 1977) is currently the most popular distance-based measure to summarize the cumulative characteristics of a spatial distribution of events in the context of micro-geographic data. It has indeed proved a very versatile tool to test for the presence of spatial concentration within a stationary point pattern where each event is considered as a dimensionless point. As a consequence, the K-function has been largely applied in various fields such as geography, ecology, epidemiology and, more recently, economics (see Arbia and Espa, 1996; Marcon and Puech, 2003).

The K-function is defined as follows:

\[ K(d) = \lambda^{-1} E \{ \text{number of points falling at a distance } \leq d \text{ from an arbitrary point} \} \]  

(1)

with \( E \{ \} \) indicating the expectation operator and \( \lambda \) representing the mean number of events per unitary area, a parameter called intensity. Therefore, \( \lambda K(d) \) can be interpreted as the expected number of further points within a distance \( d \) of an arbitrary point of the process (Ripley, 1977). In case of a homogeneous underlying field (where the probability of hosting a point is constant across the study area), the K-function quantifies the level of spatial dependence between points at each distance \( d \).

In order to develop a test for the presence of absolute spatial concentration, we can rely on the fact that for many stochastic processes, it is possible to compute the expectation in the right-hand side of Equation (1), so that \( K(d) \) can be written in a closed form (Dixon, 2002). A point process generating a spatial distribution of events completely at random (that is, points are distributed uniformly and independently on space) is the so-called homogeneous Poisson process. It can be shown that if a point pattern is a realisation of a homogeneous Poisson process then \( K(d) \) tends to be equal to \( \pi d^2 \) (see Diggle, 2003). Therefore:

\[ K(d) = \pi d^2, \ d > 0 \]

represents the null hypothesis of random location of events. Significant departures from this benchmarking value represent the alternative hypothesis of spatial dependence. More precisely, for \( K(d) > \pi d^2 \) we have positive dependence and hence clustering (where points tend to attract each other), for \( K(d) < \pi d^2 \) we have negative dependence and hence inhibition (where points tend conversely to repulse each other). Therefore, to formally test whether the observed points tend to cluster in space we can verify if, for some \( d \), \( K(d) \) is significantly greater than \( \pi d^2 \). Critical values
can be computed by Monte Carlo simulations of homogeneous Poisson processes (see Besag and Diggle, 1977).

The test for the presence of absolute concentration based on Ripley’s $K$-function, however, can be used to detect industrial agglomeration only if firms can be considered to have the same dimension. Indeed, in a context where economic activities are different in terms of dimension with the presence of small, medium and large firms, a point pattern is not a good representation of the location pattern of economic activities and, as a result, the $K$-function is no more a proper tool to summarize the spatial distribution. For instance, the simple $K$-function cannot recognize a situation like the one reported in Figure 1 as “Case 2” as a cluster. In other words, the test do not “control for the overall agglomeration of manufacturing” (Duranton and Overman, 2005).

In such a context, in order to define a proper test, we need to refer to the concepts and methods of the marked point process statistics, which is a branch of spatial statistics devoted to analyse sets of events scattered in space, where each event is not only defined by its spatial location, but also by a mark, that is a supplementary set of information which might be either quantitative or qualitative (Illian et al., 2008).

### 3 Measuring the spatial concentration of firms considering size: the mark-weighted $K$-function

#### 3.1 The mark-weighted $K$-function

The mark-weighted $K$-function, indicated as $K_{mm}(d)$, is an explorative tool proposed by Penttinen (2006) to summarize the cumulative characteristics of a homogeneous quantitative marked point pattern (that is a pattern where a quantitative mark is attached on each point). It has been proposed as a natural generalization of Ripley’s $K$-function. In order to introduce it let us first rewrite the classical $K$-function as:

$$K(d) = E \left[ \sum_{i=1}^{n} \sum_{j \neq i} I(d_{ij} \leq d) \right] / \lambda$$

where the term $d_{ij}$ is the Euclidean distance between the $i$th and $j$th arbitrary points, $n$ is the total number of points and $I(d_{ij} \leq d)$ represents the indicator function such that $I = 1$ if $d_{ij} \leq d$ and 0 otherwise. Following this notation, the mark-weighted $K$-function has a similar form but the marks are now taken into account:

$$K_{mm}(d) = E \left[ \sum_{i=1}^{n} \sum_{j \neq i} m_i m_j I(d_{ij} \leq d) \right] / \lambda \mu^2.$$  \hspace{1cm} (2)

In Equation (2) $m_i$ and $m_j$ are the marks attached to the $i$th and $j$th points, respectively, and $\mu$ is the mean of the marks. Thus the term $\lambda \mu^2 K_{mm}(d)$ can be interpreted as the mean of the sum of the products formed by the mark of the $i$th arbitrary point and the marks of all other points in the circle $d$ centred in it (Illian et al., 2008). Therefore, the mark-weighted $K$-function measures the joint cumulative distribution of marks and points at each distance $d$.

Turning now to the estimation aspects, following Penttinen (2006), a proper approximately edge-corrected unbiased estimator of $K_{mm}(d)$ is
\[
\hat{K}_{nm}(d) = \left( \sum_{i=1}^{n} \sum_{j \neq i} m_i m_j w_{ij} I(d_{ij} \leq d) \right) / n \hat{\lambda} \hat{\mu}^2
\]

where \( \hat{\lambda} = n / |A| \) is the estimated spatial intensity, \(|A|\) is the area of the study region and \( \hat{\mu} \) is the mean of the observed marks. Due to the presence of edge effects arising from the arbitrariness of the boundaries of the study region, the adjustment factor \( w_{ij} \) is introduced thus avoiding potential biases in the estimates in proximity to the boundaries of the study region. More precisely, the weight function \( w_{ij} \) expresses the reciprocal of the proportion of the area of a circle centred on the \( i \)th point, passing through the \( j \)th point, which lies within the study region \( A \) (Boots and Getis, 1988).

In an economic context, in which the marks are the values of a quantitative variable representing the firms size, the mark-weighted \( K \)-function might be used to develop a test for the presence of absolute spatial concentration. However, we need to derive the benchmark value of the function representing the null hypothesis of spatial randomness. For this reason the next paragraph is devoted to derive a stochastic model to generate marked point patterns of firms which is able to represent the stylized situations of spatial randomness and concentration in the meaning of “Case 1” (i.e. many small firms clustering in space) and “Case 2” (i.e. few large firms clustering in space).

3.2 A model for the null hypothesis of spatial randomness

The basic idea we follow is that the spatial concentration of economic activities (in the sense of “Case 1” and “Case 2”) can be originated by some form of correlation between the spatial point intensity and the marks. This would imply, for instance, that in regions characterized by high spatial point intensity the marks tend to be systematically large if such a correlation is positive or, conversely, small if such correlation is negative.

To define a model which incorporates such a correlation structure we refer to the design, already explored by Ho and Stoyan (2008), of an intensity-marked Cox process, where the spatial point intensity is driven by a Cox process and the marks are realizations of a process whose parameters are conditioned by the values of the spatial point intensity.

3.2.1 The log Gaussian Cox process for the spatial point intensity

To start with we assume that the spatial point intensity can be modelled as a log Gaussian Cox process (a specific kind of Cox process proposed by Möller et al., 1998). According to this model each generated point pattern represents a partial realization of an inhomogeneous Poisson process characterized by a spatial intensity function \( \lambda(x) \), with \( x \) representing the spatial coordinates of an arbitrary point (see Diggle, 2003). The values of \( \lambda(x) \) constitute, in turn, a realization of a positive random field \( \{\Lambda(x)\} \) such that \( \Lambda(x) = \exp\{S(x)\} \), where \( \{S(x)\} \) is a Gaussian random field with mean \( \mu_S \), variance \( \sigma_S^2 \) and correlation function \( \rho_S(d) \). \( \{\Lambda(x)\} \) is known as a log Gaussian Cox process.

The log Gaussian assumption is particularly useful because explicit expressions can be derived for the intensity and covariance structure of the point process. Indeed, according to the moment generating function of a log Gaussian distribution, the intensity \( \lambda \) of a log Gaussian Cox process \( \{\Lambda(x)\} \) can be written as:

\[
\lambda = E[\Lambda(x)] = E[\exp(S(x))] = \exp\left(\mu_S + \frac{1}{2} \sigma_S^2\right).
\]
Concerning to the covariance structure, for any arbitrary pairs of points (say \( x \) and \( x' \)),
\[
\Lambda(x)\Lambda(x') = \exp\{S(x) + S(x')\},
\]
and \( S(x) + S(x') \) is also Gaussian with mean \( m = 2\mu_s \) and variance
\[
v = 2\sigma_s^2 \left[ 1 + \rho_s (d) \right]
\]
where \( d \) is the Euclidean distance between \( x \) and \( x' \). As a result,
\[
E[\Lambda(x)\Lambda(x')] = \exp(m + v/2),
\]
and hence:
\[
E[\Lambda(x)\Lambda(x')] = \lambda^2 \exp\{\sigma_s^2 \rho_s (d)\}.
\]

3.2.2 The marks process

Our model assumes that the mark \( m(x_n) \) attached to the point \( x_n \) generated by the log Gaussian Cox process depends on the intensity of the process itself. More formally we have:
\[
m(x_n) = a\Lambda(x_n) + bE(x_n)
\]
(3)
where \( \Lambda(x_n) \) is the value of the spatial intensity at point \( x_n \) and \( E(x_n) \) is due to a residual process such that \( E(x) = \exp\{R(x)\} \), where \( R(x) \) is a Gaussian random field with mean \( \mu_R \), variance \( \sigma_R^2 \) and correlation function \( \rho_R(d) \). Thus, the expected value of process \( E(x) \), indicated with \( \varepsilon \), is
\[
\varepsilon = E[\exp\{R(x)\}] = \exp\left\{ \mu_R + \frac{1}{2} \sigma_R^2 \right\}.
\]
The two constants \( a \) and \( b \) appearing in Equation (3) are the model parameters. It is important to understand the role of these two parameters in the generation of the patterns of firms and the way in which they can model the relationship between the intensity with which firms are distributed in space and their dimension. More specifically, \( a \) is the parameter driving the correlation between the spatial point intensity process and the marks process. When \( a = 0 \) the marks are independent of the spatial intensity. Conversely when \( a > 0 \) the marks process generates marks that tend to be larger (that is larger firms) in regions characterized by a high spatial point intensity. Finally, in those cases where \( a < 0 \) the marks tend to be smaller (and hence the firms of smaller dimension) in regions characterized by a high spatial point intensity. On the other hand the parameter \( b \) represents the perturbation effect of the residual process on the correlation between marks and intensity. The larger is \( b \) in absolute value, the more the residual process disturbs the phenomenon of correlation controlled by \( a \).

The log Gaussian assumption makes the computation of the expected value of the marks process mathematically tractable, indeed we have:
\[
\mu = E[m(x)] = a\lambda \exp\{\sigma_s^2\} + b\varepsilon.
\]
It is easy to show that the expected value of the marks process would be \( a\lambda + b\varepsilon \). However, following Ho and Stoyan (2008), the true unbiased expected value is
\[
\mu = a\lambda \exp\{\sigma_s^2\} + b\varepsilon,
\]
which is larger than \( a\lambda + b\varepsilon \) when \( a > 0 \), and smaller when \( a < 0 \). For a detailed explanation of this bias correction see Ho and Stoyan (2008).

The model proposed here is particularly interesting having in mind economic application and specifically the study of firm location. In fact in the application of the present methodological framework to the problem of assessing industrial agglomeration, the marked point patterns generated when \( a = 0 \) represent the null hypothesis of spatial randomness of firms. Similarly, \( a > 0 \)

\[\text{In order to avoid any misunderstanding, note that the greek letter } E, \text{ used to indicate the residual process, and the expectation operator } E \text{ are different symbols.}\]
and $a < 0$ refer to the alternative hypothesis of spatial concentration of economic activities in the sense expressed in “Case 1” and “Case 2”, respectively, in Section 1.

To better illustrate how the model works, in the reminder of this section we will show some realizations of a marked point process. In what follows all the generated patterns are obtained using the same random seed so that all realizations are directly comparable and the differences between the patterns can be ascribed only to differences in the model parameters. Figure 2 shows the realization of the underlying spatial point intensity process given as $\Lambda(x) = \exp\{S(x)\}$ on the unit square, with mean $\mu_s = 5$, variance $\sigma_s^2 = 0.25$ and correlation function $\rho_s(d) = \exp\{-d/0.25\}$.

As we can see, in this particular realization, the spatial point intensity tends to be higher (light grey colours) towards the centre of the unitary area.

In order to illustrate the role of parameter $a$ in driving the correlation between the spatial point intensity and the marks, Figure 3 displays different realizations of the marked point process with different values for $a$. The six simulated marked point patterns appearing in Figure 3 show the net effect of parameter $a$ since $b$ is always set to zero. In each pattern the marks are rescaled to the unit interval and each point is represented by a circle with radius proportional to its rescaled mark. Figure 3 shows quite clearly that, for positive values of $a$, the marks tend to be larger where the spatial point intensity is higher, that is approximately at the centre of the unitary area (see pattern $i$, $iii$ and $v$). On the other hand, for negative values of $a$, the marks tend to be smaller where the spatial point intensity is higher (see pattern $ii$, $iv$ and $vi$). The two kind of clustering situation – namely, “Case 1” and “Case 2” – tend to be more evident when $a$ increases in absolute value.

Figure 4 shows six simulated marked point patterns with different values for $b$ which illustrate the role of this parameter in disturbing the correlation between the spatial point intensity and the marks. In all six cases the residual process $E(x)$ is characterised by $\mu_r = 5$, $\sigma_r^2 = 0.25$ and $\rho_r(d) = \exp\{-d/0.25\}$ and $a$ is set to be equal to 0.25. To understand how the parameter $b$ disturbs the effect of the parameter $a$, we can compare the patterns of Figure 4 with the pattern of Figure 3($i$) where $a = 0.25$. As $b$ increases in absolute terms, the residual process becomes relatively more important in generating the marked point patterns. In this situation the correlation between the spatial point intensity and the marks depicted by the pattern reported in Figure 2($i$) becomes less strong.

**Figure 2:** A realization of the underlying spatial point intensity (grey-scale image).

---

7 This specific form of the correlation function is known as the exponential function, see Diggle and Ribeiro (2007) for details.
Figure 3: Simulated patterns of marks according to model (3). The figure illustrates the role of parameter $a$ when $b = \text{constant} = 0$.

$i$) $a = 0.25; b = 0$

$ii$) $a = -0.25; b = 0$

$iii$) $a = 0.5; b = 0$

$iv$) $a = -0.5; b = 0$

$v$) $a = 1; b = 0$

$vi$) $a = -1; b = 0$
**Figure 4:** Simulated patterns of marks according to model (3). The figure illustrates the role of parameter $b$ when $a =$ constant $= 0.25.$

- $i) a = 0.25; b = 0.25$
- $ii) a = 0.25; b = -0.25$
- $iii) a = 0.25; b = 0.5$
- $iv) a = 0.25; b = -0.5$
- $v) a = 0.25; b = 1$
- $vi) a = 0.25; b = -1$
3.2.3 The benchmark value of the mark-weighted $K$-function

Because of the mathematical tractability of the model defined above, the corresponding theoretical mark-weighted $K$-function can be derived in a closed form. Indeed, for such a marked log-Gaussian Cox process (for $d > 0$), the mark-weighted $K$-function assumes the form:

$$K_{mn}(d) = 2\pi \int_0^d u \left[ a \lambda \exp\left(2\sigma_s^2 + 3\sigma_s^2 \rho_s(d)\right) + 2ab \lambda \exp\left(\sigma_s^2 + \frac{3}{2} \sigma_s^2 \rho_s(d)\right) \epsilon + b^2 \epsilon^2 \exp\left(\sigma_s^2 \rho_s(d)\right)\right] du$$

(4)

The formal derivation of Equation (4) is reported in the Appendix. Equation (4) above allows us to develop a test for the presence of absolute concentration of economic activities using the mark-weighted $K$-function, in which the null hypothesis of spatial randomness of firms is represented by the values of $K_{mn}(d)$ when $a = 0$. In fact, when $a = 0$, then we have:

$$K_{mn}(d) = 2\pi \int_0^d u \exp\left(\sigma_s^2 \rho_s(d)\right) du .$$

(5)

To help the visualization, Figure 5 shows the mean of $\hat{K}_{mn}(d)$ for 1000 marked point patterns generated in the unit square from model (3) with parameters $\mu_s = 5$, $\sigma_s^2 = 0.25$, $\rho_s(d) = \exp[-d/0.25]$, $\mu_R = 0$, $\sigma_R^2 = 0.25$, $\rho_R(d) = \exp[-d/0.25]$, $a = 0$ and $b = 1$. Since the theoretical function (dashed line), given by Equation (5), lies within the confidence envelopes (resulting from the highest and lowest values of $\hat{K}_{mn}(d)$ calculated from the 1000 simulations) and very close to the mean of $\hat{K}_{mn}(d)$ (solid line), the graph confirms that Equation (5) may well represent the proper benchmark to verify the presence of spatial concentration of economic activities.

Figure 5: Mean of $\hat{K}_{mn}(d)$ estimated from 1000 simulations of the marked point process following model (3) with parameters $a = 0$ and $b = 1$. The behaviour of the empirical mean is represented by the solid line. The theoretical function given by (5) is reported in the graph as a dashed line.
4. Discussion and conclusions

The spatial concentration of firms has long been a central issue in economics both under the theoretical and the applied point of view due mainly to the important policy implications. An approach to its measurement, that became recently very popular, makes use of micro data and looks at the firms as if they were dimensionless points distributed in the economic space. This approach is very attractive because it does not suffer from the problem of choosing an arbitrary partition of the economic space (such as e.g. regions, counties or countries). However in practical circumstances this is an excessive simplification since the points (firms) observed in the economic space are far from being dimensionless and are conversely characterized by different dimension measured in terms of the number of employees, the product, the capital and so on. In the literature, the papers that introduced such an approach (e.g. Arbia and Espa, 1996; Marcon and Puech, 2003) disregard the aspect of the different firm dimension and ignore the fact that a high degree of spatial concentration may result from the case of many small points clustering in definite portions of space (as it is usually considered in the literature), but also from only few large points clustering together (e.g. few large firms). In other words they are not able to distinguish between two very different issues, namely the clustering of firms and the clustering of economic activities. The aim of this paper was to introduce absolute measures of spatial concentration of firms based on an extension of Ripley’s $K$-function that accounts for the different firm dimension. In order to derive the null hypothesis of spatial randomness in this more complex environment, we developed a new stochastic model that generates marked point patterns of firms and is able to describe the various situations that could arise in empirical cases. In our model the firm dimension is expressed as a function of the spatial intensity of the point process. According to the different values assumed by the model parameters, this could result either in larger points located in areas with high intensity or, conversely, smaller points located in areas characterized by high intensity. The first case is more grounded under the economic point view where we can postulate that the same conditions that lead to a higher clustering of firms in some portions of space may also lead to the growth of the dimension of the existing firms. A good example is constituted by the action of the three Marshallian forces fostering agglomeration (Marshall, 1920). In his seminal work Marshall emphasized that industrial agglomeration can be explained by the fact that firms try to locate near suppliers to save shipping costs, by the theory of labor market pooling and by the theory of knowledge spillovers. If some of the services are internalized in one leading big company than the same forces could produce a growth of the firms’ dimension rather than an increase in the number of firms located in the area. We would expect therefore that in most practical cases the parameter $a$ in Equation (3) will be positive and large in absolute value. Similar arguments reinforcing this empirical expectation may be found in Krugman (1991).

On the basis of the stochastic model introduced here we derived the corresponding mark-weighted $K$-function and, by making use of some simulated patterns, we presented evidence that this tool represents a proper mean to detect the presence of absolute concentration of firms keeping their dimension into account.

The problem of calibrating the values of the model’s parameters in practical cases is complex and it is not undertaken here where we restricted ourselves to only the presentation of the stochastic mechanism. The inferential aspects would involve the estimation of the parameters $a$ and $b$ in Equation (3) and also of the parameters characterising the two log Gaussian processes $\Lambda(x)=\exp\{S(x)\}$ and $E(x)=\exp\{R(x)\}$ introduced in Section 3.2. A closed form for the likelihood of the model is not yet available at current state of the literature and currently the only viable possibility appears to be to exploit (as it is usual practice in such instances) a pseudo-likelihood approach as indicated in Besag (1974). We will undertake such an approach in some future work.
References


Appendix: Analytical derivation of the theoretical mark-weighted K-function

The mark-weighted K-function $K_{mm}(d)$ can be conceived as the integral of the mark correlation function $k_{mm}(d)$ (Illian et al., 2008), i.e.

$$K_{mm}(d) = 2\pi \int_0^d u k_{mm}(u) du . \ (6)$$

The mark correlation function can be given by:

$$k_{mm}(d) = \frac{E_{ot}[m(o)m(t)]}{\mu^2} \ (7)$$

where $E_{ot}[m(o)m(t)]$ denotes the conditional mean under the condition that there are points in two arbitrary locations separated by a distance $d$, which are considered as the origin $o$ and the destination $t$. $m(o)$ and $m(t)$ are the marks attached to the points located in $o$ and $t$ respectively. The term in the denominator $\mu$ represents the mean of the marks. Therefore $k_{mm}(d)$ can be interpreted as the normalized mean of the product of the marks of a pair of points separated by a distance $d$. According to Stoyan and Ho (2008), the numerator of $k_{mm}(d)$ satisfies the condition that:
\[ E_{m}[m(o)m(t)] = \frac{E[m(o)m(t)\Lambda(o)\Lambda(t)]}{E[\Lambda(o)\Lambda(t)]}. \] (8)

If \( \Lambda(x) \) is defined as in section 3.2.1 and \( m(x) \) is given by equation (3) then

\[
E[m(o)m(t)\Lambda(o)\Lambda(t)] = E\left[ a \exp\{S(o)\} + b \exp\{R(o)\}\right] E\left[ a \exp\{S(t)\} + b \exp\{R(t)\} \right] E\{S(o)S(t)\} \\
= a^2 E\left[ 2S(o) + 2S(t) \right] + ab E\left[ 2S(o) + S(t) + R(o) \right] + b^2 E\left[ R(o) + R(t) + S(o) + S(t) \right] \\
= a^2 \lambda^4 \exp\left[ 2\sigma^2 + 4\sigma^2 \rho_s(d) \right] + 2ab \lambda^3 \exp\left[ \sigma^2 + \frac{5}{2} \sigma^2 \rho_s(d) \right] \epsilon \\
+ b^2 \lambda^2 \exp\left[ \sigma^2 \rho_s(d) \right] \epsilon^2 \exp\left[ \sigma^2 \rho_r(d) \right] \\
\]

and

\[ E[\Lambda(o)\Lambda(t)] = \lambda^2 \exp\left[ \sigma^2 \rho_s(d) \right] \]

Therefore Equation (8) can be written as

\[
E_{m}[m(o)m(t)] = a^2 \lambda^2 \exp\left[ 2\sigma^2 + 3\sigma^2 \rho_s(d) \right] + 2ab \lambda \exp\left[ \sigma^2 + \frac{3}{2} \sigma^2 \rho_s(d) \right] \epsilon + b^2 \epsilon^2 \exp\left[ \sigma^2 \rho_r(d) \right] \\
\]

As a result, since \( \mu = a\lambda \exp\left[ \sigma^2 \right] + b \epsilon \), the mark correlation function has the following form:

\[
k_{mm}(d) = \frac{a^2 \lambda^2 \exp\left[ 2\sigma^2 + 3\sigma^2 \rho_s(d) \right] + 2ab \lambda \exp\left[ \sigma^2 + \frac{3}{2} \sigma^2 \rho_s(d) \right] \epsilon + b^2 \epsilon^2 \exp\left[ \sigma^2 \rho_r(d) \right]}{[a\lambda \exp\left[ \sigma^2 \right] + b \epsilon]^2}, \quad d > 0. \] (9)

Finally, by substituting Equation (9) in Equation (6) we obtain, for \( d > 0 \), the explicit form of the mark-weighted \( K \)-function:

\[
K_{mm}(d) = 2\pi \int_0^{d} u \frac{a^2 \lambda^2 \exp\left[ 2\sigma^2 + 3\sigma^2 \rho_s(d) \right] + 2ab \lambda \exp\left[ \sigma^2 + \frac{3}{2} \sigma^2 \rho_s(d) \right] \epsilon + b^2 \epsilon^2 \exp\left[ \sigma^2 \rho_r(d) \right]}{[a\lambda \exp\left[ \sigma^2 \right] + b \epsilon]^2} du \\
\] (10)