Department of Civil Engineering, Building and Environment
Geodesy and Geomatics Area

Ph.D. course “Infrastructures and Transportations”
XXIV Ciclo

VADASE
Variometric Approach for Displacement Analysis
Stand-alone Engine

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Rome, February 24, 2012
If you can keep your head when all about you
Are losing theirs and blaming it on you;
If you can trust yourself when all men doubt you,
But make allowance for their doubting too:
If you can wait and not be tired by waiting,
Or being lied about, don't deal in lies,
Or being hated, don't give way to hating,
And yet don't look too good, nor talk too wise;

If you can dream – and not make dreams your master;
If you can think – and not make thoughts your aim,
If you can meet with Triumph and Disaster
And treat those two impostors just the same:
If you can bear to hear the truth you've spoken
Twisted by knaves to make a trap for fools,
Or watch the things you gave your life to, broken,
And stoop and build 'em up with worn-out tools;

If you can make one heap of all your winnings
And risk it on one turn of pitch-and-toss,
And lose, and start again at your beginnings
And never breathe a word about your loss:
If you can force your heart and nerve and sinew
To serve your turn long after they are gone,
And so hold on when there is nothing in you
Except the Will which says to them: “Hold on!”

If you can talk with crowds and keep your virtue,
Or walk with Kings – nor lose the common touch,
If neither foes nor loving friends can hurt you,
If all men count with you, but none too much:
If you can fill the unforgiving minute
With sixty seconds' worth of distance run,
Yours is the Earth and everything that’s in it,
And – which is more – you’ll be a Man, my son!

Rudyard Kipling, If
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A.1 Differences in satellites coordinates not accounting for Sagnac effect 155
Abstract

Global Navigation Satellite Systems (GNSS), like Global Positioning System (GPS), are nowadays very well known. Besides the mass market, GPS plays an important role in several technical and scientific activities. Ever since the early stages of development (mid 1980s), given the high level of accuracy achieved in determining the coordinates of the receiver, it became clear that the extensive deployment of GPS stations all over the world would have improved many tasks in geodesy and geodynamics.

In recent years, several studies have demonstrated the effective use of GPS in estimating coseismic displacement waveforms induced by an earthquake with accuracies ranging from a few millimeters to a few centimeters (so called GPS seismology). This contribution is particularly relevant as it supports the estimation of important seismic parameters (e.g. seismic moment and magnitude $M_w$) without the problems of saturation that commonly affect seismometers and accelerometers close to large earthquakes. These studies were developed mainly off-line, analyzing observations acquired during strong earthquakes. Then, well-known processing strategies (single Precise Point Positioning (PPP), and differential positioning) have been developed to reduce as far as possible the latency between earthquake occurrence and coseismic displacement waveforms estimation.

This work describes a new approach that was originally designed to detect the 3D displacements of a single GPS receiver in real-time and that was eventually appointed as an effective strategy to contribute to GPS seismology. The main goals that guided through the development aimed to obtain the coseismic displacement waveforms in real-time and using a single receiver. In particular, it was pursued to overcome the drawbacks of the two most common strategies used so far in GPS seismology: PPP uses a single receiver, but it requires ancillary products (e.g.
informations regarding satellites orbits, clocks; parameters describing the Earth orientation with respect to an external inertial system) currently unavailable in real-time. On the other hand, differential kinematic positioning is based on a complex infrastructure (GPS permanent network), which has to be managed by specialized research centers to obtain high accuracies in real-time.

The new approach proposed in this work, named Variometric Approach for Displacement Analysis Stand-alone Engine (VADASE), is based upon a so called variometric algorithm, which is able to use a single receiver to obtain real-time results, with accuracies ranging from few centimeters up to a couple of decimeters.

In order to prove the feasibility of the variometric algorithm, a tuned software, which was appointed with the same name (i.e. VADASE), was implemented. Exploiting this software to process a large amount of both simulated and real data, the main advantages of VADASE, as well as its limitations, were investigated in details. At a first stage, VADASE effectiveness was proven on a simulated example. First, a known displacement (1 cm East and North and 2 cm Up) was synthetically introduced into carrier phase observations collected at 1 Hz rate by the M0SE permanent GPS station (Rome, Italy). Then, these data were processed using the variometric algorithm: the displacements were estimated with an accuracy of 1 ÷ 2 mm in the horizontal and the vertical directions. The solutions obtained using the GPS broadcast products available in real-time and the best quality products supplied by International GNSS Service (IGS) a posteriori displayed a global agreement of 1 mm for the horizontal components and 2 mm for the height.

Given this fundamental proof of success, and provided that the variometric algorithm was originally conceived to contribute in the field of GPS seismology and tsunami warning systems, VADASE was applied to retrieve the coseismic displacements and the waveforms generated by real earthquakes. Here, it is worth underlining that all VADASE results were obtained using exclusively broadcast products that are available in real-time. The most significant outcomes were obtained by considering data collected at 1 Hz rate from the IGS station of BREW during the Denali Fault, Alaska earthquake ($M_w$ 7.9, November 3, 2002), at 10 Hz rate from CADO station during the L’Aquila earthquake ($M_w$ 6.3, April 6, 2009), and at 5 Hz rate from some stations included in the University NAVSTAR Consortium (UNAVCO)-Plate Boundary Observatory (PBO) network during the Baja, California earthquake ($M_w$ 7.2, April 4, 2010). In all cases, the agreement between VADASE and other solutions achieved by different research groups employing different methodologies was between few centimeters and a couple of decimeters.

The real-time potentialities of the variometric approach were internationally
recognized during the recent tremendous earthquake in Japan ($M_w = 9.0$, March 11, 2011), when the GNSS research team of “Sapienza” University of Rome was able to provide the first waveforms results among the scientific community. The results obtained for IGS stations of MIZU, USUD and from EV-network station JA01 were compared with those stemming from the PPP approach implemented in the software developed at Natural Resources Canada (NRCan) [62]. The agreement between VADASE and PPP was evaluated in terms of the Root Mean Square Error (RMSE) of the differences. In addition, it was investigated the agreement reliance with the earthquake duration. The RMSE of the differences between the two solutions ranged from $0.01 \div 0.02$ m in East and North and $0.06$ m in Up, after one minute, to approximately $0.05$ m in East and North and (with much larger variability) $0.20$ m in Up after four minutes. Further, the agreement between the two approaches evaluated in terms of peak to peak displacements appeared to be independent from the earthquake duration. In details, the differences reached approximately $0.01$ m in East and North and $0.03$ m in Up components. Finally, the correlation coefficient between the two solutions proved to be higher than $99\%$ for the planimetric components (but for the East component of USUD and the North component of JA01 which showed $97\%$ and $96\%$ correlations, respectively). The vertical component showed slightly lower (and with larger variability) correlations: $65\%$, $84\%$ and $90\%$ for stations MIZU, USUD and JA01, respectively.

At the moment of this writing, VADASE is subject of a pending patent of the “Sapienza” University of Rome ever since June 2010. In October 2010 VADASE was recognized as a simple and effective approach towards real-time coseismic displacement waveform estimation and it was awarded the German Aerospace Agency (DLR) Special Topic Prize and the First Audience Award in the European Satellite Navigation Competition (ESNC) 2010.

\[1\] This work was made possible thanks to the precious support of Dr. Henton Joe and Dr. Dragert Herb, NRCan, who produced the PPP solutions for the aforementioned stations.
Chapter 1

Introduction

1.1 Overview

Global Navigation Satellite Systems (GNSS), like Global Positioning System (GPS), are nowadays very well known. Even though, GPS was originally set up by the United States (US) Department of Defence (DoD) for military purposes, many people currently use it as an embedded tool in their car and/or mobile satellite navigator. Besides the mass market, GPS plays an important role in several technical and scientific activities. With no sake of completeness it is worth to mention the support to positioning and mapping activities around the world, the studies of tectonic movements (millimeter accuracy) of the Earth and the investigations of the atmospheric structure (e.g. troposphere and ionosphere).

Ever since the early stages of development (mid 1980s), given the high level of accuracy achieved in determining the coordinates of the receiver, it became clear that the extensive deployment of GPS stations all over the world would have improved many tasks in geodesy and geodynamics. At that time, the observations were typically acquired every 30 seconds (or with a lower rate) and the data were combined together to achieve one position solution per day. These solutions were then stacked in time series of coordinates and, as a matter of fact, they revealed as an invaluable tool to monitor long-period large-scale geophysical and geodynamical events such as crustal deformation, sea-level changes, post-glacial crustal rebound and coseismic and postseismic deformations. On the other hand,
the much larger, even though temporary, displacements produced by the elastic waves radiated from the earthquake source could not be observed due to the low data acquisition rate and the low temporal resolution of coordinates estimation.

A meaningful breakthrough came in the mid and late 1990s, when the advances achieved in GPS receiver technology, together with the increased data storage capability, generated the possibility to acquire and store satellite observations with much higher (up to 20 Hz) sampling rates. As a consequence, conventional geodetic models were upgraded in order to analyze data at high sampling rates ($\geq 1$ Hz) and to solve for the receiver position at every observation epoch. This combination of events can be considered as the birth of GPS Seismology, which can be thought of as employing high-rate GPS data and solving for the real-time kinematic positions of the receiver.

In recent years, several studies have demonstrated the effective use of GPS in estimating coseismic displacement waveforms induced by an earthquake with accuracies ranging from a few millimeters to a few centimeters. This contribution is particularly relevant as it supports the estimation of important seismic parameters (e.g. seismic moment and magnitude $M_w$) without the problems of saturation that commonly affect seismometers and accelerometers close to large earthquakes. These studies were developed mainly off-line, analyzing observations acquired during strong earthquakes. Then, well-known processing strategies (single Precise Point Positioning (PPP), and differential positioning) have been developed to reduce as far as possible the latency between earthquake occurrence and coseismic displacement waveforms estimation.

1.2 The main contributions of the present research

In this framework, this work describes a new approach that was originally designed to detect the 3D displacements of a single GPS receiver in real-time and that was eventually appointed as an effective strategy to contribute to GPS seismology. The main goals that guided through the development aimed to obtain the coseismic displacement waveforms in real-time and using a single receiver.

In particular, it was pursued to overcome the drawbacks of the two most common strategies used so far in GPS seismology: PPP uses a single receiver, but it requires ancillary products (e.g. informations regarding satellites orbits, clocks; parameters describing the Earth orientation with respect to an external inertial system) currently unavailable in real-time. On the other hand, differential kinematic positioning is based on a complex infrastructure (GPS permanent network), which has to be managed by specialized research centers to obtain high
accuracies in real-time.

The new approach proposed in this work, named Variometric Approach for Displacement Analysis Stand-alone Engine (VADASE), is able to use a single receiver to obtain real-time results, with accuracies ranging from few centimeters up to a couple of decimeters. Moreover, the algorithm does not require any powerful hardware and can be directly embedded in the receiver firmware. Simple transmission equipment can be added to allow communication if a defined displacement’s threshold is (being) exceeded. With such a configuration, in case an earthquake occurs, the waveform can be retrieved in real-time with centimeter accuracy and immediately transmitted to a remote control center, which can decide whether or not to put out a tsunami alert. Up to now, however, VADASE has been implemented in a preliminary software release, able to work with GNSS observations and ephemerides as input files. This step was necessary to test its functionality and to confirm the low computational burden of the algorithm.

At the moment of this writing, VADASE is subject of a pending patent of the “Sapienza” University of Rome ever since June 2010. In October 2010 VADASE was recognized as a simple and effective approach towards real-time coseismic displacement waveform estimation and it was awarded the German Aerospace Agency (DLR) Special Topic Prize and the First Audience Award in the European Satellite Navigation Competition (ESNC) 2010. VADASE’s effectiveness was internationally recognized during the recent tremendous earthquake in Japan (March 11, 2011 Mw 9.0), when GNSS research team of “Sapienza” University of Rome was able to provide the first waveforms results among the scientific community.

1.3 Outline

The present work is organized in 7 chapters. The first one is meant as a general introduction in order to give an insight into the applications of GPS in supporting seismology. Chapter 2 can be divided into two main parts. At the beginning, a brief history of the rapid developments that interested GNSS technology up to recent years together with a summary of the main features characterizing each GNSS are given. In the second part, some of the services provided to the user community by International GNSS Service (IGS) are presented and the main observables broadcast by satellites and used in data processing are exposed. To conclude, section 2.5 provides some details about satellites orbit computation using both broadcast and precise GNSS ephemerides.

The main contributions given by GNSS to geophysics are reported in the first section of Chapter 3. Here, the concept of GPS seismology is introduced and the advantages and limitations that using GNSS receivers can bring about with
1.3. Outline

respect to classical seismology instruments (accelerometers and velocimeters) are discussed. Importantly, it is given a brief review of the main applications that benefit from the high-rate GNSS data availability, with a closer look to earthquake and tsunami warning systems. To conclude, the advantages of the brand new approach presented in this work are highlighted in comparison with the most used strategies in GPS seismology.

Chapter 4 presents the functional and the stochastic models of the variometric algorithm that was designed and implemented within this work. In this regard, it is initially shown how the carrier phase observations are combined to form the variometric observations. Then, the least squares estimation problem is described in details. Section 4.2 describes the application of the variometric algorithm to retrieve the displacements synthetically imposed to a stream of real data coming from M0SE, Rome, Italy GPS permanent station. In conclusion, a simulated example shows how losses of lock of the signals and cycle slips result as outliers and can be easily recognized and excluded from the observations using a tuned Data Snooping (DS) technique.

Chapter 5 presents all the results achieved from the analysis of the data simulated with a Spirent GNSS simulator, owned by DLR. Most of the work described in this chapter is the outcome of a three month period expended at DLR in the framework of the present Ph.D. research. Overall, chapter 5 permits highlighting the different terms that are encompassed in the variometric model and gives a marked understanding of its potentialities and limitations. In a first example, the signals are simulated as traveling with no disturbances along the path from the satellite to the receiver. In this case, the capability of the variometric algorithm to work with Galileo constellation is shown and some related issues are discussed. Then, all the terms that cause range biases (e.g. orbit errors, atmospheric contributions, satellite clock errors) are assessed by means of a tuned simulation. Further, the effectiveness of VADASE in retrieving a known motion imposed to the receiver is reported. In addition, a sensitivity analysis shows that an initial error in the receiver coordinates would cause a drift in the retrieved waveforms. Finally, three simulations are specifically experienced to test the variometric algorithm with respect to rapid changes in the atmospheric conditions. The effects of these events over the estimated receiver displacements are discussed and the basis of a statistic technique to be used in order to address the mentioned perturbations is described.

The most important results obtained applying the variometric algorithm to the analysis of real data recorded during earthquake events are reported in chapter 6. In details, several earthquakes are considered (e.g. Denali Fault, Alaska earthquake $M_w$ 7.9, November 3, 2002; L’Aquila earthquake $M_w$ 6.3, April 6, 2009; Baja, California earthquake $M_w$ 7.2, April 4, 2010). In each case, the receiver waveforms are retrieved and visually compared with the solutions ob-
tained by other research groups adopting different approaches (i.e. either PPP or instantaneous positioning). Fundamentally, for the great Japan earthquake, March 11, 2011 the variometric approach was the first to allow the coseismic displacement and waveforms determination. For this case, a detailed comparison of VADASE solutions with respect to PPP solutions provided by Natural Resources Canada (NRCan) is assessed and some comments about the major limitations of the variometric algorithm are discussed. The present thesis ends with chapter 7 which summarizes the reasons of this work, the main results achieved and the issues to be further investigated.
1.3. Outline
Chapter 2

Global Navigation Satellite Systems

GNSS is a set of artificial satellite constellations which are nowadays used for countless different applications. Generally speaking, these systems were designed to allow the instantaneous determination of position and velocity (i.e. navigation) and time of a generic user receiving signals broadcast by satellites which are set in orbit around the Earth.

The former system to be developed and deployed was the US NAVigation Satellite Time and Ranging (NAVSTAR) GPS. It was originally set up by the DoD of US for military purposes. In a first stage, to prevent real-time use of GPS by non U.S. Army (including real-time civilian use), the GPS signals were scrambled by artificial satellite clock dithering (Selective Availability (S/A)) in such a way that only the military were able to fully exploit the system. Things changed on May 1st 2000, when U.S. president Bill Clinton decided to turn off S/A by “pushing a button”, tearing down the position accuracy from 100 meters to less than 10 meters and giving birth to commercial development of GPS.

The wide success of GPS has led to the development of other similar systems operated from different countries. The ensemble of such systems is referred to as GNSS. The main reason for the development of alternative systems to GPS is to ensure access to GNSS signals that are not under the control of any single nation, with implications for the military in times of war and national emergencies, and for civilian institutions that have stringent requirements on guaranteed access to a sufficient number of GNSS signals at all times [13].
Today, GNSS industry is estimated to be worth billions of dollars, almost any new smart-phone or car has an embedded GNSS receiver (that costs just a few hundreds or cents (u-blox bulk) of dollars) for navigation purposes and there are countless Information Systems (Google Earth, Microsoft Virtual Earth, . . . .) that describe reality within a global reference frame that GNSS contributed to establish.

Besides the mass market, GNSS plays an important role in several technical and scientific activities. With no sake of completeness it is worth to mention the support to positioning, navigation and mapping activities around the world, the studies of tectonic movements (millimeter accuracy) of the Earth and the investigations of the atmospheric structure (e.g. troposphere and ionosphere) [24].

2.1 A brief history

Looking at the present state of the art of the satellite technology and thinking about how our lives are leveraged by satellites applications, someone could be impressed to imagine that everything started by just over fifty-five years ago. It was October 4, 1957 and the first man-made satellite, namely the Sputnik I, was set to orbit around the Earth by the Soviet’s Union government. Sputnik I was just a metallic sphere with four long antennas and contained a built in transmitter to broadcast signals to Earth.

Few days after the launch, two scientists from the John Hopkins Applied Physics Laboratory (APL), Guier and Weiffenbach, succeeded in receiving and recording complete passes of the satellite from horizon to horizon with no modulation on the 20 MHz frequency, which they would later call pure Doppler shift [37]. Within few days, and profiting also from the signals broadcast from Sputnik II, they realized that a complete set of Keplerian orbit parameters for a near Earth satellite could be inferred to useful accuracy from a single set of Doppler shift data (so called “direct problem”). More importantly, inverting the problem, the position of the receiver could be determined while assuming the orbit known.

This knowledge led, in a very short time, to designing the essentials of the complete Transit Navigation System: multiple polar orbiting satellites radiating two stable frequencies encoded with their orbit parameters, a satellite tracking system receiving these same two frequencies to solve the direct problem, and an injection station to transmit the resulting orbit parameters to each satellite, which would continue to orbit the Earth so that military means with navigation receivers/computers could determine their position about once an hour anywhere on Earth [37]. Since no signals were emitted from the receiver side, Transit was
a complete “passive” system, able to provide the user position without revealing it to the enemy. Transit system was developed in the first 1960s by the US military and, after an initial experimental phase [27], the first operational satellite was launched on December 5, 1963 and the first position measurement took place in 1964. Although the system was conceived for military purposes, when the deployment was complete (i.e. six satellites in nearly circular polar orbits), in 1967, it was released to the public and was used by oceanographers, fishing fleets, and oil exploration companies [64]. Each satellite transmitted two carrier frequencies (to eliminate the ionosphere disturbance effect) at 150 and 400 MHz and the position accuracies achievable with dual frequency receivers were on the order of about 20 meters [42, p. 5]. In addition, the height determination was so poor that the system could be be used to achieve only two-dimensional positions (i.e. latitude and longitude of the receiving station).

Following the success of Transit, which is no longer operative ever since 1996, Soviet’s Union developed a similar system called Tsikada (or Cicada) trying to shorten the gap with the United States, who at the same time were already moving forward aiming to improve the achievable accuracies and to reduce the time needed for positioning.

In April 1964, Naval Research Laboratory (NRL) scientist Rogerl L. Easton formulated a concept for transmitting ranging signals such that the distance to the target satellite could also be measured, making early orbit determination. This idea led to the concept of a new generation satellite-based navigation system, called TIMATION, which would have used ranging from satellites. The TIMATION Project, under the direction of Roger L. Easton, concentrated on developing and improving quartz frequency standard for satellites and determining the most effective satellite constellation for providing worldwide coverage [60].

In the same period (i.e. 1963-1964), Aerospace Corporation started to conceive a new system, later denoted as System 621B, employing satellites to improve navigation for fast moving vehicles in three-dimensions. The main requirements taken into account while planning this system aimed to obtain “a true navigational system . . . with an unlimited number of users . . . providing global coverage . . . sufficiently accurate to meet military needs” [63, p. 22]. The trade studies conducted by Aerospace Corp. showed a concept that provided a high-dynamic capability using two Pseudorandom Noise (PRN) signals. Aircraft accuracy was demonstrated to be less than 5 m for position and less than 0.3 m/sec for velocity [63].

At that point, to avoid uncontrolled proliferation of individually pursued pro-
2.1. A brief history

Programs concerning satellite systems, the Deputy Secretary of Defense took action to combine all the different activities with a lead service (the Joint Program Office (JPO), which was to be located at the Space and Missile System Organization (SAMSO) at Los Angeles Air Station) in order to minimize development and productions costs.

After this major direction, in 1973, the JPO was instructed by the U.S. DoD to establish, develop, test, acquire, and deploy a spaceborne positioning system which could gather all the know-how acquired with the different programs undertaken at that time (e.g. Transit, TIMATION, System 621B) [64, p. 3]. The first satellite of the fresh born NAVSTAR GPS program was launched in 1978 and further developments led to have a Full Operational Capability (FOC) in 1995. At that time, 24 GPS satellites were operational in their assigned orbits and the constellation was tested for military performances [42, p. 310].

As a matter of fact, GPS was the first satellite system to reach the full operational capability and for a long period of time it has been by far the most used among the civilian users and the scientific community. Although the Russian counterpart GLObal’naya NAVigatsionnaya Sputnikovaya Sistema (GLONASS) reached for the first time its FOC with 24 satellites in 1996, the system downgraded immediately because of the strong economic crisis that hit Russia. In fact, since no new satellites were launched until 1999 and because of the reduced lifetime of GLONASS satellites with respect to GPS ones (3 years compared to the 7.5 years), the number of GLONASS operational satellites rapidly decreased to less than 10.

Nowadays, things have totally changed and the list of global and regional satellite-based positioning systems displays a continuous growth. New GNSS, such as the European Galileo or the Chinese Compass, are going to join the already existing GPS and GLONASS constellations. Moreover, the Russian system is again fully operational with 24 active satellites ever since October 31, 2011 [70]. In addition, these systems are supplemented by Space-based Augmentation Systems (SBAS) (e.g. Wide-area Augmentation Systems (WAAS), European Geostationary Navigation Overlay Service (EGNOS) or Ground-based Augmentation Systems (GBAS). A detailed description of the current status of the satellite-based navigation systems can be found in [31] (and references therein). In the next section, the fundamental parts of a global navigation satellite system are briefly described following the approach in [42, pp. 6-7].

\footnote{The first two Galileo satellites were launched October 21, 2011. The full operational capability is foreseen for 2020 for both Galileo and Compass}
2.2 GNSS elements

Each GNSS consists of three different segments: the Space segment, the Control segment and the User segment.

Space segment

In order to provide a continuous global positioning capability, each GNSS system should develop a satellite constellation which ensures that (at least) four satellites are always electronically visible everywhere on the Earth. The design of the orbit has to follow various optimization criteria which take into account the service coverage, the required user position accuracy, the satellite geometry, ...

GNSS satellites are equipped with atomic clocks, radio transceivers, computers and other ancillary material used for normal satellite operation. The signals sent from space down to the user allow to measure the biased range (pseudorange) to the satellite. Further, each satellite broadcasts a message which allows to compute satellite position for arbitrary instants. The main source of power is supplied by solar panels. A small propulsion system permits to maneuver the satellite in order to adjust its orbit.

The satellites have several systems of identification:

- the launch sequence number
- the orbital position number
- the international designation

Control segment

The control segment is responsible for steering the whole system [42]. This encompasses the deployment and the maintenance of the system, tracking of satellites for the determination and the prediction of orbital and clock parameters, and uploading the data message to the satellites using ground antennas. A master station controls the activities of all the other tracking stations (e.g. GPS master control station is located at Schriever Air Force Base, near Colorado Springs, Colorado, whereas GLONASS master control station is located in Moscow).

User segment

This segment encompasses all the users which employ the system to different extents. Military and Army organizations have access to the system up to its full capability. This is not the case for civilian and unauthorized users which, for instance, do not have access to all signals broadcast by the satellites. Several
private and governmental services have been established to deliver GNSS status information, data and products to the user. Here, it is worth mentioning the IGS [29] which is committed to providing the highest quality data and products as the standard for GNSS in support of Earth science research, multidisciplinary applications, and education. Currently the IGS includes two GNSS, GPS and the Russian GLONASS, and intends to incorporate future GNSS.

2.3 The International GNSS Service

The International GNSS Service was officially established in January 1994 as a service of the International Association of Geodesy (IAG). The initial name of the organization, which was International GPS Service, was changed to the actual one in 2005 to acknowledge the fact that its activities encompass multi Global Navigation Satellite Systems.

IGS operates as a voluntary, non-commercial confederation of about 200 institutions world-wide (a map of the global stations configuration is shown in figure 2.1).

Figure 2.1: Global distribution of IGS network stations - http://igscb.jpl.nasa.gov/network/netindex.html

The mission of the IGS is “to provide the highest-quality GNSS data and products in support of the terrestrial reference frame, Earth rotation, Earth ob-
2.4. GNSS observables

Observation and research, positioning, navigation and timing and other applications that benefit society” [29].

At the basis of IGS success there is the culture of data sharing: a large set of GNSS data (GPS/GLONASS pseudorange and phase observations, broadcast ephemerides, and supporting types of raw data (such as meteorological)) and products (precise orbit and clock files) are freely available via the Internet from the IGS Global Data Center. To facilitate extensive data exchange a set of standard file formats has been established (table 2.1).

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Table 2.1: Formats currently used by IGS - http://igscb.jpl.nasa.gov/components/formats.html

The infrastructure has nowadays become quite complex due to the large number of participating institutions. To efficiently organize and assess the different aspects of current GNSS products generation a number of working groups have been created. Additionally, several pilot projects are investigating the future developments which could lead to the generation of new IGS products. For the sake of this work it is worth mentioning the Real-Time Pilot Project (RTPP) which aims, among others, to manage and maintain a global IGS tracking GNSS network, to distribute observations and derived products to real-time users, to generate new real-time products and to investigate standards and formats for real time data collection, data dissemination and delivery of derived products. More recently, the IGS has been following the development of the European Galileo system [30].

2.4 GNSS observables

This section aims to expose the basic principles of GNSS positioning and to review some of the most important aspects of the observation equations for pseudoranges and carrier phases. Generally speaking, and keeping in mind the differences
between various GNSS, it is possible to say that all satellite transmissions are
derived from a fundamental frequency which is made available by onboard atomic
clocks. Any further detail about signal generation and modulation would be
system dependent and is out of the scope of this work. For a detailed description
of the signal structure of the Global Navigation Satellite System used in the
present work (i.e. GPS, GLONASS and Galileo) it is possible to refer to [44, 35,
32].

Pseudoranges and carrier phases are the most important GNSS observables.
Different solutions for positioning are available using pseudoranges only, carrier
phases only, or combinations of both types of observations. Overall, the satellite-
based positioning concept is founded on the principle of “trilateration”, which is
the method of determining position by measuring distances to points of known
position [13, p. 363]. In the case of GNSS, the known points would be the
positions of the satellites in view and the distances could be measured as the
time difference between the receiver local clock and the atomic clock onboard a
satellite multiplied by the speed of light.

2.4.1 Code pseudorange

The pseudorange between a generic satellite $s$ and a generic receiver $r$ is mea-
sured as the time difference between the epoch of signal transmission, tagged by
the internal satellite clock ($t^s$), and the epoch of signal reception, tagged by the
internal receiver clock ($t^r$). This difference is then multiplied by the speed of light
($c$) to obtain a measure in length units (meters). Practically speaking, the satel-
lite sends its clock time by multiplying the carrier phase by a known sequence
of $+1$ and $-1$ (i.e. the so called “pseudorandom code”). The receiver internally
generates an identical replica of the code and performs a cross-correlation with
the incoming signal to compute the time shift necessary to align the two codes.
This time difference, multiplied by the speed of light, gives the pseudorange mea-
surement.

By assuming a reference common time system for the clock onboard the satel-
lite and the receiver clock (i.e. for $t^s$ and $t^r$, respectively), it is unavoidable that
both clocks suffer a delay with respect to the common time system. The pseu-
dorange observation equation can be obtained considering, at first, that signals
are traveling in the vacuum

$$P^s_{r,i}(t_r) = c (t_r - t^s) = c (t_r + \delta t_r - t^s - \delta t^s) \quad (2.1)$$

where $P^s_r$ is the pseudorange between the satellite and the receiver, $i$ is the
frequency of the carrier phase used to modulate the code, $t_r$ is the time tag of
the receiver clock, $\delta t_r$ is the delay of the receiver clock from the reference time
2.4. GNSS observables

system (i.e. receiver clock error), \( t^s \) is the time tag of the satellite clock, \( \delta t^s \) is the delay of the satellite clock from the reference time system (i.e. satellite clock error), and \( c \) is the speed of light.

Now, since \( t_r \) and \( t^s \) refer to the same time system, the difference \( \Delta t = t_r - t^s \) is the true signal travel time and, if multiplied by the speed of light, returns the geometric distance \( \rho^s(t^s) \) between the position of the satellite at epoch \( t^s \) and the position of the receiver at epoch \( t_r \):

\[
\rho^s_r(t^s) = \sqrt{(X^s - X_r)^2 + (Y^s - Y_r)^2 + (Z^s - Z_r)^2}
\]

where \( X^s, Y^s, Z^s \) and \( X_r, Y_r, Z_r \) are the coordinates of the satellite and of the receiver, respectively. Hence, the final pseudorange equation for a signal propagating in the vacuum reads as follows

\[
P_{r,i}^s(t_r) = \rho^s_r(t^s) + c(\delta t_r - \delta t^s) \tag{2.2}
\]

2.4.2 Carrier phase

The carrier phase observable results from the difference between the phase of the incoming carrier wave (upon which the code is modulated) and the phase of a signal internally generated by the receiver which is synchronized with the receiver clock [13, p. 367]. If the recording starts at epoch \( t_0 \), the receiver is capable to measure only the fractional part of the carrier phase that arrives from the satellite. Hence, the integer number of cycles between the satellite and the receiver at the initial epoch \( t_0 \) (the so called “integer ambiguity” \( N \)) is unknown. In unit of cycles the carrier phase observable reads as follows

\[
\phi^s_{r,i}(t_r) = \frac{f_i}{c} \rho^s_r(t^s) + N^s_{r,i} + f_i \delta t_r - f_i \delta t^s \tag{2.3}
\]

where \( i \) is the frequency of the carrier wave, \( \phi^s_{r,i} \) is the phase observation between satellite \( s \) and receiver \( r \), \( c \) is the speed of light, \( \rho^s_r \) is the geometric distance between the position of the satellite at epoch \( t^s \) and the position of the receiver at epoch \( t_r \), \( N^s_{r,i} \) is the integer ambiguity and \( \delta t_r \) and \( \delta t^s \) are the receiver clock and the satellite clock errors, respectively. The carrier phase can be scaled to length unit multiplying equation 2.3 by the wavelength of the carrier phase \( i \), \( \lambda_i = c/f_i \):

\[
L^s_{r,i}(t_r) = \rho^s_r(t^s) + \lambda_i N^s_{r,i} + c(\delta t_r - \delta t^s) \tag{2.4}
\]

It is worth to underline that the integer ambiguity \( N^s_{r,i} \) refers to the first epoch of observations \( t_0 \) and remains constant during the period of observations if the tracking of the satellite is continuous (i.e. without loss of lock). Hence, the generic carrier phase observation at epoch \( t \) is given by

\[
\phi^s_{r,i}(t) = \Delta \phi^s_{r,i} \bigg|_{t_0} + N^s_{r,i} \tag{2.5}
\]
2.4. GNSS observables

where $\Delta \phi^{s}_{r,i}$ is the (measurable) fractional part of the carrier phase at epoch $t$ augmented with the integer number of cycles passed from the initial epoch $t_0$. This number is initially set to zero when the receiver is turned on and the first phase observations is recorded. Then, it is incremented by one cycle whenever the fractional phase changes from $2\pi$ to 0 [42, p. 194]. In case a loss of lock of the signal occurs, this number is reset to zero causing a jump in the accumulated phase observation. This jump is referred to as cycle slip.

Equation 2.4 could be read as a biased distance to the satellite. In fact, this type of pseudorange is about 100 times more precise than the code pseudorange. However, the major drawback of the carrier phase observation stands in the integer ambiguity, which should be resolved exactly (using the so called “ambiguity resolution” techniques) to achieve the best possible accuracy in geodetic positioning.

2.4.3 Measurement delays

Up to now, the signals have been considered to travel in the vacuum. In addition, the electronic noises due to the receiver and the satellite hardwares have been neglected together with other systematic effects. A possible list of systematic biases that affect the signals encompasses the satellite orbit errors, the satellite clock errors, the propagation effects, the receiver clock errors, the relativistic effects, the antenna phase center variations, and the multipath. Some of these biases can be modeled and result in additional terms in the observations equation. On the other hand, some systematic effects can be eliminated (or strongly reduced) by applying appropriate combinations of the observables (i.e. the ionospheric delay).

To consider these terms in the pseudorange and the carrier phase observations it is necessary to revise equations 2.2 and 2.3, including (some) systematic and random biases, as follows

\[
P^{s}_{r,i}(t_r) = \rho^{s}_{r}(t^s) + c (\delta t_r - \delta t^s) + T^{s}_{r}(t_r) + I^{s}_{r,i}(t_r) +
+m^{s}_{r,i}(t_r) + \epsilon^{s}_{r,i,P} +
\]

\[
L^{s}_{r,i}(t_r) = \rho^{s}_{r}(t^s) + \lambda_i N^{s}_{r,i} + c (\delta t_r - \delta t^s) + T^{s}_{r}(t_r) - I^{s}_{r,i}(t_r) +
+m^{s}_{r,i}(t_r) + \epsilon^{s}_{r,i,L} +
\]

where $T^{s}_{r}(t_r)$ is the tropospheric delay, $I^{s}_{r,i}(t_r)$ is the ionospheric delay, $m^{s}_{r,i}$ is the multipath and $\epsilon^{s}_{r,i,P}$ and $\epsilon^{s}_{r,i,L}$ are terms that encompass the noises. Recalling that the additions in the pseudoranges and carrier phase observations are limited to the delays that can be considered relevant for the present work, a brief description of the added biases is given hereafter:
2.4. GNSS observables

\( T^s_{tr}(t_r) \)  
Tropospheric delay. It is due to the neutral (i.e. non ionized) part of the Earth’s atmosphere. This region extends up to 50 km from the surface and it is the region in which most of the weather-related phenomena occur. The tropospheric delay depends upon the tropospheric conditions along the ray path and it is independent of the frequency of the carrier (this applies up to frequencies of 15 GHz). This results in an immediate drawback: it is not possible to eliminate the tropospheric delay combining observations from two different frequencies. Instead, different models are available to account for the tropospheric delay. Saastamoinen and STANAG models, which are implemented within the new algorithm proposed within this work, are exposed at pages 37 and 78, respectively.

\( I^s_{tr,i}(t_r) \)  
Ionospheric delay. It is due to the free electrons in the Earth’s atmosphere (in particular in the ionosphere region, which extends from 70 km up to 1000 km above the Earth). The ionosphere is a dispersive medium with respect to GNSS signals. Therefore, a linear combination of two frequencies can lead to elimination (up to the second order terms) of the ionospheric delay. It is worth to underline that the ionospheric delay causes a delay in the pseudorange measurement whereas the carrier phase is advanced. This effect, known as the ionospheric divergence, is due to the difference of phase and group refractive indexes and is exposed in more details at page 83.

\( m^s_{tr,i}(t_r) \)  
Multipath effect. It is caused by signals which reach the receiver not on their direct path, but after being reflected from the nearby environment. This effect can be significantly reduced selecting sites protected from reflections and adopting appropriate design for the receiving antenna. In addition, since the multipath effect is frequency dependent, its influence can be estimated using a combination of pseudoranges and carrier phases measurements on different frequencies [42, p. 154].

\( \epsilon^s_{tr,i,P/L} \)  
Hardware and observations noises. It is dependent on the wavelength \( (\lambda_i) \) of the signal \( (10^{-2} \div 10^{-3}\lambda_i) [42, \text{p. 198}] \) and on the satellite and receiver hardware.

2.4.4 Observation differences

Usually, to reduce or eliminate some of the systematic effects described above, it is possible to form differences using pseudorange and carrier phase observations described in equations 2.6 and 2.7. Even though biases’ elimination has the advantage that the number of unknowns reduces significantly, as a direct
2.4. GNSS observables

consequence no estimations for the eliminated parameters will be available (e.g. satellite and receiver clocks errors). In this section, the single and double differences will be presented for pseudorange and carrier phase observables.

**Single-differences**

The single difference is a new observable stemming from the linear combination of two measurements. Generally speaking, it is possible to form at least three kinds of single differences, namely:

- **receivers single-differences**, using the simultaneous measurements of the same satellite from two different receivers. In this case, the satellite clock error $\delta t^s$ is (almost) eliminated
- **satellites single-differences**, using the simultaneous measurements of two satellites from the same receiver. Here, the receiver clock error $\delta t^r$ is eliminated
- **epoch single-differences**, using the observations from the same receiver and the same satellite at two different epochs. If there is no loss of lock of the signal, the initial integer ambiguity $N^s_{r,i}$ is eliminated

For the sake of the present work, it is important to describe the single-difference that is formed using two different receivers and one satellite. In section 4.1 it will be explained how the epoch single-differences are used in the innovative variometric algorithm that is presented in this work. For two receivers, $r$ and $l$, observing the same satellite $s$ at the nominal times $t_r$ and $t_l$, it is possible to write two pseudorange equations $2.6$ and two carrier phase equations $2.7$. Considering that the receivers are observing simultaneously, the epochs $t_r$ and $t_l$ are equal. Nonetheless, the respective signals leave the satellite $s$ at slightly different epochs, because the distances between receivers and satellite differ [53, p. 175]. However, because of the high stability of the satellites’ atomic clocks, it is possible to assume that the satellite clock error is the same for the two transmissions. The carrier phase single-difference displays as follows

$$L^s_{r,l,i}(t) = L^s_{l,i}(t) - L^s_{r,l}(t) =$$

$$= \rho^s_{r,l}(t^s) + \lambda_i N^s_{r,l,i} + c(\delta t^l - \delta t^r) + T^s_{r,l}(t) - I^s_{r,l,i}(t) +$$

$$+ m^s_{r,l,i}(t) + \epsilon^s_{r,l,i,L}$$

where the subscripts notation is intended to operate as follows

$$\tau^s_{r,l}(t) = \tau^s(t) - \tau^s(t)$$

It is worth to notice that the satellite clock error and the satellite hardware delay have canceled in the single-difference observation.
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Double-differences

Now, if two receivers \( r \) and \( l \) observe two satellites \( s \) and \( u \) simultaneously, two single-differences observations \( L_{rl}^s(t) \) and \( L_{rl}^u(t) \) (equation 2.8) are combined to form the double-difference \( L_{rl}^{su}(t) \), which displays as follows

\[
L_{rl,i}^{su}(t) = L_{rl,i}^s(t) - L_{rl,i}^u(t) =
\rho_{rl}^{su}(t) + \lambda_i N_{rl,i}^{su} + T_{rl,i}^{su}(t) - I_{rl,i}^{su}(t) + m_{rl,i}^{su}(t) + \varepsilon_{rl,i}^{su}(2.9)
\]

where the superscript and subscript notation is intended to operate as follows

\[
\cdot_{rl}(t) = (\cdot_{rl}(t) - \cdot_{rl}(t)) - (\cdot_{rl}(t) - \cdot_{rl}(t))
\]

The most notable feature of equation 2.9 is that the large receiver clock errors \( \delta_{lr} \) and \( \delta_{lt} \) have canceled. The double-difference is the basic observation used by several GNSS softwares (e.g. Bernese [26]) for precise parameters estimation.

2.4.5 Linear combinations

Linear combinations of the original carrier phase or pseudorange observations are often used in order either to eliminate systematic biases (e.g. ionosphere-free combination) or to facilitate parameters estimation (e.g. geometry free combination to estimate the ionospheric models). These linear combinations can be formed using as input any differencing level of the original observations (i.e. from zero-difference observations up to double-difference observations). The main drawback of the linear combinations is that the noise of the observable is higher than the original ones. The formulation of the generic linear combination reads as follows

\[
S_{LC} = \alpha S_i + \beta S_j
\]

where \( \alpha \) and \( \beta \) are the real coefficients of the combination and \( L_i \) and \( L_j \) represent the original observations (either the pseudorange or the carrier phase) in length units and with frequencies \( i \) and \( j \), respectively. The noise of the linear combination can be expressed as

\[
\sigma_{LC} = (\sqrt{\alpha^2 + \beta^2}) \sigma_0
\]

with the assumption that both the original observations have the same level of noise \( \sigma_0 \).

For a broader insight of the different linear combinations that are mostly used in GNSS data processing it is possible to look at [26, pp 39-41]. For the interest of this work it is sufficient to report the ionosphere-free combination, which is used by the brand new variometric algorithm described in section 4.1.
2.4. GNSS observables

Ionosphere-free linear combination

The ionosphere-free combination is the most efficient method to eliminate the ionospheric delay by using two signals with different frequencies. This method can be thought of as the main reason for the GNSS signals to be broadcast with (at least) two different frequencies. The ionosphere-free combination is formed from equation 2.10 using the following coefficients

\[
\alpha = \frac{f_i^2}{f_i^2 - f_j^2} \quad \beta = \frac{-f_j^2}{f_i^2 - f_j^2}
\]  

(2.12)

which have the property that \( \alpha + \beta = 1 \). To show the effectiveness of this method in removing the ionospheric delay, let write the carrier phase observation equation 2.7 with the explicit notation of the dependency of the ionospheric delay on signal frequency

\[
L_{s,r,i}^s = \rho_{r}^{s} + \lambda_{i} N_{r,i}^s + c(\delta t_{r} - \delta t^{s}) + T_{r}^s - \frac{A \cdot TEC_{r}^s}{f_i^2}
\]  

(2.13)

introducing the Total Electron Content (TEC) along the ray path from the satellite to the receiver, with \( A = 40.3 \ [m^3 \ s^{-2}] \). In equation 2.13, the epoch tag, the multipath and the noise terms have been eliminated for the sake of simplicity.

Now, the ionosphere-free combination for two carrier phase observations with frequencies \( i \) and \( j \) is formed as follows

\[
L_{r,iF}^s = \alpha L_{r,i}^s + \beta L_{r,j}^s = \rho_{r}^{s} + N_{r,iF}^s + c(\delta t_{r} - \delta t^{s}) + T_{r}^s +
\]

\[
\left( \frac{f_i^2}{f_i^2 - f_j^2} \cdot \frac{-A \cdot TEC_{r}^s}{f_i^2} \right) + \left( \frac{-f_j^2}{f_i^2 - f_j^2} \cdot \frac{-A \cdot TEC_{r}^s}{f_j^2} \right)
\]  

(2.14)

where the acronym \( IF \) stands for Ionosphere-Free and \( \lambda_{IF} \) is the wavelength of the new formed observable.

As equation 2.14 shows, the ionosphere-free combination cancels the ionospheric delay. The major drawback of this combination is that the bias due to the ambiguity is no longer an integer value, as it is displayed by the following equations

\[
N_{r,IF}^s = \frac{1}{f_i^2 - f_j^2} (f_i^2 \lambda_{i} N_{r,i}^s - f_j^2 \lambda_{j} N_{r,j}^s)
\]  

(2.15)

\[2\]Here, we give a first approximation of the ionosphere dependency upon the frequency, stopping at the first order term and neglecting higher terms
2.5 Satellite orbits

All satellite applications (from navigation to Earth observation) require satellite orbits to be known with a certain level of accuracy. For positioning purposes, in case a single receiver is used, any orbital error is highly correlated with the position error. On the contrary, in the case of a baseline estimation between two receivers, relative orbital errors are approximately equal to relative baseline errors [42].

Generally, the orbital motion of a satellite is the result of all forces acting on the satellite, the Earth’s gravitational attraction being the most significant one. From the mathematical point of view, the equations of motion for satellites are differential equations that are solved by numerical integration over time. For a complete dissertation on theory and practice of satellites orbits determination and prediction it is wise to refer to [56].

GNSS positions are available to the users in real-time as a specific part of the satellite message: the so called “broadcast ephemerides” are a set of parameters which allows the orbit computation with an accuracy ranging from one meter up to several meters, according to the considered constellation (i.e. GPS, GLONASS, Galileo, ...). Broadcast ephemerides are predicted using the observations acquired at the monitoring stations of the respective control segment. These parameters effectively describe satellite orbits for a short period of time; then, they rapidly become inaccurate.

More precise orbital information (up to centimeter accuracy level) are available to the users after a certain period of time (latency) with respect to satellite message acquisition. These are the so called “precise ephemerides” and consist on American Standard Code for Information Interchange (ASCII) files listing satellite positions and clock errors every fifteen minutes. Precise ephemerides can be publicly accessed from web repositories maintained by research centers (e.g. Jet Propulsion Laboratory (JPL), Center for Orbit Determination in Europe (CODE), ...) or international organizations (e.g. IGS).

For the scopes of this work it is sufficient to describe the algorithms used to compute GNSS orbits, employing either the “broadcast” or the “precise” ephemerides. These methods have been included in the implementation that was performed as a part of this work in order to test the so called “variometric” approach (presented in section 4.1). The discussion is limited to satellite systems implemented and tested with the VADASE software at the moment of writing: GPS, GLONASS and Galileo.
2.5. Satellite orbits

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_0$</td>
<td>Mean (M.) Anomaly (Reference Time)</td>
<td>semi-circles (sc)</td>
</tr>
<tr>
<td>$\Delta n$</td>
<td>M. Motion Diff. From Computed Value</td>
<td>sc/sec</td>
</tr>
<tr>
<td>$e$</td>
<td>Eccentricity</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$\sqrt{A}$</td>
<td>Square Root of the Semi-Major Axis</td>
<td>meters</td>
</tr>
<tr>
<td>$\Omega_0$</td>
<td>Longitude of Ascending Node (weekly)</td>
<td>sc</td>
</tr>
<tr>
<td>$i_0$</td>
<td>Inclination Angle at Reference Time</td>
<td>sc</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Argument of Perigee</td>
<td>sc</td>
</tr>
<tr>
<td>$\dot{\Omega}$</td>
<td>Rate of Right Ascension</td>
<td>sc/sec</td>
</tr>
<tr>
<td>$IDOT$</td>
<td>Rate of inclination angle</td>
<td>sc/sec</td>
</tr>
<tr>
<td>$C_{rc}, C_{rs}$</td>
<td>Corr. Terms (C.T.) – Orbit Radius</td>
<td>meters</td>
</tr>
<tr>
<td>$C_{uc}, C_{us}$</td>
<td>C.T. – Argument of Latitude</td>
<td>radians</td>
</tr>
<tr>
<td>$C_{ic}, C_{is}$</td>
<td>C.T. – Angle of Inclination</td>
<td>radians</td>
</tr>
<tr>
<td>$t_{oe}$</td>
<td>Reference Time for Ephemeris</td>
<td>seconds</td>
</tr>
<tr>
<td>$IODE$</td>
<td>Issue of Data (Ephemeris)</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2: GPS and Galileo Keplerian parameters in broadcast orbits

2.5.1 Broadcast ephemerides

The broadcast ephemerides are based upon the observations acquired by the monitoring stations of the respective control segment. The master station is entrusted with the ephemerides computation and uploading to the satellites. Generally, the broadcast ephemerides contain records with general information about the satellite, records with orbital information and records with information on the satellite clock.

GPS and Galileo

As regards GPS and Galileo constellations the orbit information contained in the broadcast ephemerides encompass the orbital Keplerian parameters and their temporal variation. At the same time, the information on the satellite clock is given in the terms of coefficients that can be used to model the clock offset with polynomials of second order. Based on broadcast ephemerides, whose parameters are described in table 2.2, GPS and Galileo coordinates computation is performed using the algorithm described in table 2.3 [44, 32].
It should be underlined that \( t \) is the GPS/Galileo system time at epoch of transmission (i.e. GPS/Galileo time corrected for time of flight). \( t_k \) should be the actual total time difference between the time \( t \) and the epoch time \( t_{oe} \) and must account for beginning or end of weeks crossovers.

It is worth noting that, although the algorithm used to compute satellite position is the same, GPS and Galileo have different reference frames for coordinates and time. This difference, which is not addressed in details in this work (refer to [32, pp. 46-48] and the list of recommendations [78, pp. 78-83]), should be taken into account when observations of both systems are employed for navigation or positioning purposes.

Satellite clock error at observation epoch \( t \) can be computed applying the following second order polynomial

\[
\delta t^s(t) = a_0 + a_1 (t - t_{oe}) + a_2 (t - t_{oe})^2 + \Delta t_r
\]

(2.16)

where \( a_0 \) [s] is the satellite clock bias, \( a_1 \) [s/s] is the satellite clock drift, \( a_2 \) [s/s^2] is the satellite frequency drift, \( t_{oe} \) [s] is the clock data reference time, \( \Delta t_r \) [s] is the correction due to relativity effects and \( t \) is the current time epoch. More details about the satellite clock error computation for GPS and Galileo are given in section 5.3.

GLONASS

The GLONASS broadcast navigation message is generally transmitted as a half-hourly satellite state vector, expressed in the PZ90 geocentric cartesian coordinate system [76]. Ephemeris parameters are periodically computed and uploaded to the satellites by the control segment. Mean square errors of transmitted coordinates and velocities of the satellites are given in table 2.4.

Given the state vector in the broadcast ephemeris at epoch \( t_b \), satellite positions and velocities for the generic epoch \( t_0 \), with \( |t_b - t_0| \leq 15 \) min, can be computed by numerical integration of the differential equation of motion. Integration interval shortness leads to the simplification of the force model by neglecting perturbing forces acting on the satellite but the lunar-solar effect which is assumed to be constant.

Without going through all the intermediate steps (which are duly reported in [35, 76]), the final form of the satellite’s equations of motion in the PZ90 system is given
2.5. Satellite orbits

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Ellipsoid</td>
<td>WGS84</td>
</tr>
<tr>
<td>Numerical Constants</td>
<td></td>
</tr>
<tr>
<td>( \mu ) = 3.986005 \cdot 10^{14} , [m^3 , s^{-2}]</td>
<td>Gravitational Constant</td>
</tr>
<tr>
<td>( \dot{\Omega}_e ) = 7.2921151467 \cdot 10^{-5} , [rad , s^{-1}]</td>
<td>Earth Rotation Rate</td>
</tr>
<tr>
<td>( \pi ) = 3.1415926535898</td>
<td></td>
</tr>
</tbody>
</table>

**Computation formulas**

\[

t_k = t - t_{oe} \quad \text{Time from Ephemeris Epoch}
\]

\[
A = \left( \sqrt{\frac{\mu}{A^3}} \right) \quad \text{Semi-Major Axis}
\]

\[
n_0 = \frac{\sqrt{\mu}}{A^3} \quad \text{Computed Mean (M.) Motion}
\]

\[
n = n_0 + \Delta n \quad \text{Corrected (Corr.) M. Motion}
\]

\[
M_k = M_0 + n t_k \quad \text{M. Anomaly (An.)}
\]

\[
E_k = M_k + esinE_k \quad \text{Eccentric An. (by iteration)}
\]

\[
\nu_k = tan^{-1}\sqrt{1-e^2sinE_k} \quad \text{True An.}
\]

\[
u_k = \omega_0 + \nu_k \quad \text{Argument of Latitude (Lat.)}
\]

\[
\delta u_k = C_{uc}\cos2u_k + C_{us}\sin2u_k \quad \text{Argument of Lat. Correction}
\]

\[
\delta r_k = C_{rc}\cos2u_k + C_{rs}\sin2u_k \quad \text{Radius Correction}
\]

\[
\delta i_k = C_{ic}\cos2u_k + C_{is}\sin2u_k \quad \text{Inclination Correction}
\]

\[
\omega_k = \omega_0 + \delta u_k \quad \text{Corr. Argument of Perigee}
\]

\[
r_k = A(1 - ecosE_k) + \delta r_k \quad \text{Corr. Radius}
\]

\[
i_k = i_0 + it_k + \delta i_k \quad \text{Corr. Inclination}
\]

\[
x_k = r_k\cos(\omega_k + \nu_k) \quad \text{x-coordinate – Orbital Plane}
\]

\[
y_k = r_k\sin(\omega_k + \nu_k) \quad \text{y-coordinate – Orbital Plane}
\]

\[
\Omega_k = \Omega_0 + \dot{\Omega} t_k - \Omega_e(t - t_0) \quad \text{Corr. Ascending Node Long.}
\]

\[
X_k = x_k\cos\Omega_k - y_k\sin\Omega_k\cos i_k \quad \text{ECEF Sat X Coordinate}
\]

\[
Y_k = x_k\sin\Omega_k + y_k\cos\Omega_k\cos i_k \quad \text{ECEF Sat Y Coordinate}
\]

\[
Z_k = y_k\sin i_k \quad \text{ECEF Sat Z Coordinate}
\]

Table 2.3: GPS and Galileo satellite coordinates computation
## 2.5. Satellite orbits

<table>
<thead>
<tr>
<th>Error component</th>
<th>Mean square error coordinates [m]</th>
<th>Mean square error velocity [cm/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Along track</td>
<td>7.0</td>
<td>0.03</td>
</tr>
<tr>
<td>Cross track</td>
<td>7.0</td>
<td>0.03</td>
</tr>
<tr>
<td>Radial component</td>
<td>1.5</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 2.4: Accuracy of transmitted coordinates and velocities of GLONASS-M satellites [35]

\[
\ddot{x} = -\frac{\mu}{r^3} x - \frac{3}{2} J_0^2 \frac{\mu a_e^2}{r^5} x \left(1 - \frac{5 z^2}{r^2}\right) + \omega^2 x + 2 \omega \dot{y} + \ddot{x}_{S+M} \tag{2.17}
\]

\[
\ddot{y} = -\frac{\mu}{r^3} y - \frac{3}{2} J_0^2 \frac{\mu a_e^2}{r^5} y \left(1 - \frac{5 z^2}{r^2}\right) + \omega^2 y - 2 \omega \dot{x} + \ddot{y}_{S+M} \tag{2.18}
\]

\[
\ddot{z} = -\frac{\mu}{r^3} z - \frac{3}{2} J_0^2 \frac{\mu a_e^2}{r^5} z \left(3 - \frac{5 z^2}{r^2}\right) + \ddot{z}_{S+M} \tag{2.19}
\]

where \((\ddot{x}, \ddot{y}, \ddot{z})_{S+M}\) are the acceleration due to the lunar-solar gravitational perturbations and remain constant during the integration interval, \(\mu = 398600.44 \cdot 10^9 [m^3/s^2]\) is the gravitational constant, \(a_e = 6378136 [m]\) is the semi-major axis of the Earth, \(J_0^2 = 1082625.7 \cdot 10^{-9}\) is the second zonal harmonic of the geopotential and \(\omega = 7.292115 \cdot 10^{-5} [rad/s]\) is the Earth rotation rate. The initial conditions for the integration of reduced equations set are the components of the satellite state vector at epoch \(t_b\) \((x(t_b), y(t_b), z(t_b), \dot{x}(t_b), \dot{y}(t_b), \dot{z}(t_b))\). The integration is performed using 4th order Runge-Kutta technique (a good reference for Runge-Kutta implementation is given in [66, pp. 710-715]).

### 2.5.2 Precise ephemerides

Precise ephemerides consist of ASCII files containing satellite positions, velocities and clock errors at equidistant epochs (typical time spacing is 15 minutes). These ephemerides can be publicly accessed from web repositories and are available to the user with a certain latency with respect to satellite signal acquisition. The higher the latency (up to 17 days) the higher the level of accuracy of the precise ephemerides (up to a few centimeters). An overview of the precise products and the related accuracies delivered by one of the outstanding international organization such as the IGS can be found in [43]. Currently, IGS precise ephemerides are free for all users and are exchanged by means of the standard SP3 format [75].
2.5. Satellite orbits

To obtain the position and the velocity vectors for the generic epoch $t$ that lies between the given epochs it is possible to use interpolation. The most used interpolation techniques are Lagrange interpolation, based on polynomial base functions, and the trigonometric interpolation that exploits the periodic nature of the GNSS orbits. A detailed comparison between the strengths and the weaknesses of the two methods is given in [71]. In the present work, to obtain satellites positions at the generic epoch $t$ using precise ephemerides, Lagrange interpolation technique has been chosen due to its speed and its smooth implementation procedure.

Assuming that the functional values $f(t_j)$ are given at time intervals $t_j$, $j = 0, 1, \ldots, n$, then the approximated value of $f$ at epoch $t$ results from

$$
\sum_{j=0}^{n} f(t_j) a_j(t)
$$

(2.20)

where

$$
a_j = \frac{(t - t_o) (t - t_1) \cdots (t - t_{j-1}) (t - t_{j+1}) \cdots (t - t_n)}{(t_j - t_o) (t_j - t_1) \cdots (t_j - t_{j-1}) (t_j - t_{j+1}) \cdots (t_j - t_n)}
$$

is the Lagrange operator and has the following properties

$$
a_j(t) = \begin{cases} 
1 & \text{for } t = t_j \\
0 & \text{otherwise}
\end{cases}
$$

so that $a_j(t_j) = f_j$, which means that Lagrange operator returns the exact value of $f$ at the given epoch.
Chapter 3

High-rate data and GPS seismology

3.1 GPS contribution to geophysics

Space geodetic techniques, such as Lunar Laser Ranging (LLR), Very-Long Baseline Interferometry (VLBI), Satellite Laser Ranging (SLR) and GNSS, have been deployed since the early 1970s with the aim of overcoming classical terrestrial surveying methods limitations and to define a global reference frame [13]. In particular, since the early 1990s GPS proved to be capable of providing global stations coordinates and velocities in a common global reference frame with centimeter and millimeter level accuracy, respectively [28, 49, 17].

The most used processing method was the so called relative (or differential) positioning. The key observation was the double difference (see section 2.4.4).

In this approach, the combination of carrier phase observations from two receivers is used to cancel out common GPS satellite and receiver clock errors and to reduce other error sources in order to determine the baseline between the occupying stations. Then, assuming that the coordinates of one station (so called reference) are known, the user position can be inferred. The most immediate constrain is that the observations should be simultaneously recorded at the reference and the unknown station(s).
The main source of error that was limiting the accuracy of the differential positioning for long baseline or large networks was the orbit error [11]. The rough rule of thumb formulated in [9] states that the error in the baseline estimation is a direct function of the orbit error and that it grows with the baseline length, as follows

\[ \Delta \rho_{\text{coord}} \approx \frac{l}{d} \cdot \Delta \rho_{\text{orbit}} \]  

(3.1)

where \( \Delta \rho_{\text{orbit}} \) is the error in satellite position (orbit error), \( \Delta \rho_{\text{coord}} \) is the induced error in the coordinates, \( l \) is the approximate length of the baseline and \( d \) is the approximate distance between the receivers and the GPS satellites. Hence, using broadcast orbits (\( \Delta \rho_{\text{orbit}} \approx \frac{2}{3} \) m) to estimate a baseline with \( l \approx 1200 \) km would yield to \( \Delta \rho_{\text{coord}} \approx 0.1 \div 0.2 \) m (considering \( d \approx 25000 \) km).

Given such considerations, it resulted very clear that using broadcast orbits was severely limiting positioning accuracy and that huge efforts were to be put on a large scale in improving satellites coordinates determination. Therefore, in 1994 it was created the International GPS Service (IGS) with the important aim of producing and making available with a time latency of 10 \( \div \) 15 days accurate (at early stages, Root Mean Square Error (RMSE) \( \approx 0.25 \) m) satellite orbits, referred to as precise or final orbits. A wider discussion about IGS precise products and about the multiple activities undertaken by the organization in supporting Earth sciences is given in [11].

As a first consequence of the advent of precise orbits the errors in differential positioning coming from the space segment were virtually eliminated [47]. Further, exploiting the high accuracy level of the new precise products (satellite orbits and clocks) it was developed another positioning approach, referred to as PPP, to be used as an alternative to differential positioning.

In this approach, undifferenced dual-frequency pseudorange and carrier phase observations, along with precise orbit products, are used to solve for the position of a single receiver with centimeter level accuracy. The major drawback of relative positioning (i.e. having simultaneous observations available at two GPS stations) is overcome thanks to the satellite clock estimates available with the satellite coordinates in the precise orbit products. In addition, other systematic effects that cause centimeter variations in the satellite to user range, and whose impact is less significant when the observation were differentiated, are carefully modeled. For a broader discussion about the mentioned effects it is possible to refer to [47].

At present, both the described approaches are implemented in the most used...
3.2. GPS contribution to seismology

Ever since the early stages of development, given the high achieved level of accuracy, it became clear that the extensive deployment of GPS stations all over the world would have improved many tasks in geodesy and geodynamics. At that time, the observations were typically acquired every 30 seconds (or with a lower rate) and the data were combined together to achieve one position solution per day. These solutions were then stacked in time series of coordinates and, as a matter of fact, they revealed as an invaluable tool to monitor long-period large-scale geophysical and geodynamical events such as crustal deformation, sea-level changes, post-glacial crustal rebound and coseismic and postseismic deformations.

For the first time, observations coming from an array of continuous operating GPS receivers were used to measure the coseismic displacement caused by the $M_w$ 7.2 Landers, California earthquake in 1992 [16, 18]. Here, it has to be noted that with the aforementioned data collection strategy it was only possible to model the coseismic displacement as a step function in the solution time series. On the other hand, the much larger, even though temporary, displacements produced by the elastic waves radiated from the earthquake source could not be observed due to the low data acquisition rate and the low temporal resolution of coordinates estimation [61].

In the mid and late 1990s, the important advances achieved in GPS receiver technology, together with the increased data storage capability, generated the possibility to acquire and store satellite observations with much higher (up to 20 Hz) sampling rates. At this point, much interest arose in widening GPS fields of application to include using the receiver as a seismometer to represent the waveforms caused by large magnitude events. To this aim, some experiments were specifically executed by the Disaster Prevention Research Institute (DPRI), Kyoto University, Japan, to assess the ability of high-rate (1 Hz) GPS receivers to sense large amplitude periodic signals, such those that could be observed during an earthquake [41].

In 1999, similar experiments with further encouraging results were executed.
3.2. GPS contribution to seismology

by Ge [33], University of New South Wales, Sidney, Australia. He used two Leica CRS1000 GPS receivers operating in Real-Time Kinematic (RTK) mode with a fast sampling rate of 10 Hz to test the feasibility of using a GPS receiver as a seismometer in order to directly measure large displacements [33]. One of the receivers was located on a mechanical shaker capable of generating vibrations of given amplitudes and frequency, the other was located 160 km away. In addition, an accelerometer was co-located on the vibrating GPS antenna to provide an independent measurement from comparison. In details, the shaker vibrated with frequencies of 2.3 Hz and 4.3 Hz and displacements up to 12.7 mm and the oscillations were recognized in both time and frequency domain for both the horizontal and the vertical components of the RTK time series. In addition, the GPS RTK solutions were compared with the accelerations integrated twice. The comparison showed a very good agreement within the intervals when the vibrations were operated by the shaker.

Overall, these experiments demonstrated that with fast sampling rates (up to 20 Hz) GPS could be used as a seismometer for measuring displacements directly [34]. From a practical standpoint, this can be considered as the birth of GPS Seismology, which can be thought of as the application of conventional geodetic models in analyzing GPS data at high sampling rates (≥ 1 Hz) [50] and solving for the receiver position at every observation epoch.

At this point, one could ask which are the advantages that GPS introduces in the seismology field in which other well-known instruments, such as velocimeters and accelerometers, are already commonly and consistently in use. To answer this question it is useful to recall the major differences and similarities of the two instruments.

To begin with, inertial seismometers are electromechanical systems that measure the relative motions between an internal mass (which is suspended) and the instrument frame [61]. Such systems produce signals (either velocities or accelerations, according to the instrument type) when they are directly perturbed. On the other hand, GPS receivers record the pseudorange measurements to the satellites and the position of the antenna should be estimated from these ranges [12]. Hence, a first advantage of GPS instruments is that they are not prone to saturation or tilting, which, instead, commonly affect seismometers that are close to large magnitude earthquakes.

Naturally, a disadvantage of GPS instruments is that their sensitivity to seismic ground motion is not nearly as good as that of seismometers. In this respect, Larson [50] suggested that GPS could most effectively contribute to seismology if used as a strong motion instrument. In fact, displacements recovered after

---

1This term is used by seismologists who study large amplitude ground motions
the integration of seismometers signals usually display large drifts, thus limiting the accuracy in measuring coseismic displacements caused by strong earthquake. This is not the case for GPS instruments: they return the receiver displacements as primary result, without any need of integration, and, therefore, can be consistently used to measure large displacements with high accuracy.

Further, a notable difference lies in the reference system used by the two instruments: seismometers record direct signals in an inertial reference frame whereas using GPS it is possible to recover positions or displacements with respect to a global terrestrial reference frame.

To conclude, it appears clear that both instruments have advantages and drawbacks and that each of them can better serve to highlight and assess different aspects related to seismic events. As an important adding remark, it is worth noting that the extensive deployment of continuously operating GPS receivers contributes to cover areas of interest where the seismometers are not present at all (or vice versa). Overall, the contribution of GPS can be seen as an effective push forward to seismology towards the global coverage and the better assessment of large earthquakes and seismic events.

### 3.3 The benefits of high-rate GPS data

New technologies and analysis methods are providing access to GPS data and products with increasing sampling rate and decreasing latency, thus broadening the fields of applications of such instruments [38].

With no sake of completeness, it is worth mentioning some examples of large networks that dispense high rate GPS observations. At present, ∼ 240 GPS stations of the EarthScope Plate Boundary Observatory (PBO) [83], western United States, and other sites encompassed in the California Real Time Network (CRTN) [73] are being upgraded to provide 1 Hz data at better than 0.3 seconds latency. Moreover, the National Aeronautics and Space Administration (NASA) Global Differential GPS (GDGPS) project delivers streams of positions at 1 Hz for over 120 globally distributed stations [58]. The Japanese GPS Earth Observation Network (GEONET) consists of 1200 permanent GPS stations, with an average interdistance of about 20 kilometers, which transmit observations at 1 Hz rate to a central control station that is entrusted to store and archive the data and to execute the network near real-time processing.

From a scientific point of view, the added value of having high-rate low latency data derives from the possibility to enhance the temporal resolution in the observations of natural processes in Earth systems [38]. Further, low latency data
3.3. The benefits of high-rate GPS data

delivery to a remote control center allows to assess and coordinate rapid scientific responses to natural hazards. At the same time, it prevents from accidental data losses due to receiver destruction or possible following failures in the data communication infrastructure.

Many fields of research will benefit from this groundbreaking availability of data (e.g. [38]). As regards the present work, however, it is sufficient to underline how high-rate GPS observations will enhance the assessment and the understanding of seismic events (i.e. GPS Seismology) and how this contribution is likely to impact tsunami warning systems.

Firstly, as already discussed in section 3.2, the integration of GPS and seismometer data series can improve the determination of the seismic source. In fact, many deformation processes related to the earthquake cycle do not cause waves (i.e. it can not be observed by classical seismological instruments) and, generally, take place over a wide range of time scales.

Moreover, static displacements measured with GPS can be used to improve the rapid determination of the earthquake slip model and the related surface deformation forecasts. This effectively enhance the characterization of the event since the early stages. In this respect, the high quality of earthquake source and shaking model determination returned by GPS is fundamental. Various investigations have been undertaken in order to combine geodetic and seismic data sets to constrain the coseismic slip distribution [68, 67]. Other researches showed how the real-time GPS data play a fundamental role in the realization of Earthquake Early Warning (EEW) systems [21, 2, 15].

Finally, the development of early tsunami warning systems strongly relies on the fast and accurate determination of the magnitude, the propagation direction and the motion of the sea floor [82]. Displacements at GPS sites can be used to infer the motion of the sea floor by constraining a fault slip model.

More significantly, for the Sumatra 2004 earthquake and the successive tsunami Blewitt [14] showed that if GPS data had been processed and understood in real-time, the true magnitude of the event could have been estimated in less than 15 minutes. For the same event, Sobolev [72] demonstrated that the inversion of the slip distribution could have been resolved with near-field (< 100 km) GPS sites. As a consequence, the earthquake magnitude, which at the beginning was underestimated relying only on the few seismological data available in real-time, could have been correctly addressed approximately 10 minutes after the event, thus giving the possibility to raise a tsunami warning.
3.4 Processing strategies used for GPS Seismology: Pros and Cons

Given the technology developments (high acquisition rate, large data storage capability) and the possibility to solve for the receiver position at every observation epoch, GPS data have been used in monitoring seismic waveforms and determining the coseismic offsets since the early 2000s [48, 51, 20, 65, 14, 12, 52].

Up to date, the following two approaches have been adopted for GPS Seismology: single Precise Point Positioning (PPP) and differential positioning. Coseismic displacements have been estimated with a post-processing approach using various scientific softwares (e.g. Bernese, GAMIT, GIPSY), employing high quality products (orbits, clocks, Earth Orientation Parameters (EOPs)) supplied by the International GNSS Service (IGS) or by NASA JPL. These products are freely available and have a latency period ranging from 17-41 hours (the so called rapid products) to 12-18 days (the so called precise or final products). Even at present, these products are not routinely available with the appropriate high quality in real-time.

Additionally, the long latency, which prevents real-time coseismic displacements estimations, has been overcome by the technique referred to as Instantaneous Positioning [19]. This technique is based on differential positioning and allows the fundamental resolution of integer cycle phase ambiguities using single epoch dual-frequency carrier phase and pseudorange observations. Hence, Instantaneous Positioning provides a precise independently computed position for each observation epoch, at the sampling rate of the receiver.

Even if this technique is able to guarantee high, real-time accuracy (at 1 cm level), it only provides a relative coseismic displacement, which is the displacement with respect to (at least) one reference station due to the adopted differential positioning approach. In addition, to achieve the mentioned accuracy of 1 cm in real-time, Instantaneous Positioning requires both a complex and continuously linked infrastructure (GPS permanent network) with a maximum average inter-station distance of up to several tens to a few hundreds of kilometers.

Common processing of collected data is operated in a centralized analysis center, a serious limitation for strong earthquakes that may involve the entire area covered by the GPS permanent network, including reference station(s). For such a case, coseismic displacements in a global reference frame are not available any more, since even the reference station/stations undergoes/undergo a displacement. On the other hand, for realistic natural hazard applications, one cannot assume a very large GPS permanent network since:

- the accuracy in estimating the relative position of an unknown receiver
3.5. The present challenges and the brand new variometric approach

with respect to a known reference station degrades along with the baseline length

- economic considerations limit the number of stations that can be installed and managed together in a unique infrastructure

Overall, GPS seismology has been proven to be an effective tool but requires either high quality products (orbits, clocks, EOPs) to obtain an a posteriori highest accuracy estimation of the coseismic displacements within the global reference frame, or a complex and continuously linked infrastructure in order to obtain real-time, highly accurate (1 cm level) but only relative, coseismic displacements.

3.5 The present challenges and the brand new variometric approach

Considering that very high-rate (up to 100 Hz) GPS measurements are now commonly available, a new and important contribution of GPS Seismology is the real-time earthquake source determination that, as it was briefly discussed in section 3.3, can also contribute to tsunami early warning.

In this regard, during the Real Time GPS Science Requirements Workshop held in September, 2007 in Leavenworth (Washington, USA), the goal of achieving 1 cm real-time GNSS coseismic displacement accuracies in the global reference frame, within the three minutes following an earthquake, was adopted [15].

Following these recommendations, this work proposes a new approach for estimating coseismic displacements in the global reference frame in real-time. The approach is based on a single GPS station technique able to overcome some of the difficulties displayed by the two aforementioned, presently adopted, approaches for GPS Seismology (see section 3.4). This approach is named Variometric Approach for Displacement Analysis Stand-alone Engine (VADASE) and is based on a so called “variometric” solution that requires only the standard GPS broadcast products (orbits and clocks) and the observations collected by a unique, stand-alone, dual frequency GPS receiver.

Since VADASE does not require either additional technological complexity or a centralized data analysis, in principle it can be embedded into the GPS receiver firmware, thereby providing a significant contribution to tsunami early warning systems.
Chapter 4

**VADASE algorithm**

4.1 The variometric approach and its estimation model

In this section, the basic idea of VADASE algorithm and the related estimation model are presented. The first step to start with to describe the so called “variometric” algorithm is the standard raw carrier phase observation equation [42], which in length unit reads

\[
\lambda \Phi_{r}^{s} = \rho_{r}^{s} + c(\delta t_{r} - \delta t^{s}) + T_{r}^{s} - I_{r}^{s} + \lambda N_{r}^{s} + p_{r}^{s} + m_{r}^{s} + \epsilon_{r}^{s}
\]  

(4.1)

subscript \((r)\) refers to a particular receiver and superscript \((s)\) to a satellite. \(\Phi_{r}^{s}\) is the carrier phase observation of the receiver with respect to the satellite; \(\lambda\) is the carrier phase wavelength; \(\rho_{r}^{s}\) is the geometric range (i.e. the distance between the satellite and the receiver); \(c\) is the speed of light; \(\delta t_{r}\) and \(\delta t^{s}\) are the receiver and the satellite clock errors, respectively; \(T_{r}^{s}\) and \(I_{r}^{s}\) are the tropospheric and ionospheric delays along the path from the satellite to the receiver, respectively; \(N_{r}^{s}\) is the initial phase ambiguity; \(p_{r}^{s}\) is the sum of the other effects (relativistic effects, phase center variation, phase wind-up); and \(m_{r}^{s}\) and \(\epsilon_{r}^{s}\) represent the multipath and the noise, respectively. To be fully consistent with notation, it is necessary to underline that all the terms of equation 4.1, but the initial phase ambiguity, are related to observation epoch \((t)\). Here, for the sake of simplicity,
both the time and the frequency dependency are omitted.

If we consider dual frequency GNSS observations free from cycle slips and if we differentiate equation 4.1 in the time between two consecutive epochs \((t, t+1)\), the ionospheric term may be canceled to the second order by applying the ionosphere-free combination to the time single-difference, which becomes the ionosphere-free time single-difference equation

\[
\alpha[\lambda \Delta \Phi^s_r(t, t+1)]_{L1} + \beta[\lambda \Delta \Phi^s_r(t, t+1)]_{L2} = \Delta \rho^s_r(t, t+1) + \frac{c}{2} (\Delta \delta t_r(t, t+1) - \Delta \delta t^s_r(t, t+1)) + \Delta T^s_r(t, t+1) + \Delta m^s_r(t, t+1) + \Delta \epsilon^s_r(t, t+1)
\]

where \(\alpha = (f^2_{L1}/(f^2_{L1} - f^2_{L2})\) and \(\beta = (-f^2_{L2}/(f^2_{L1} - f^2_{L2})\) are the standard coefficients of the ionosphere-free combination, \(\Delta m^s_r(t, t+1)\) and \(\Delta \epsilon^s_r(t, t+1)\) are the multipath and the noise in the time single-difference, respectively (note that while using this convention the value of \(t\) is always an integer and represents the time of observation in units equal to the inverse of the observation collection rate).

At first, if we hypothesize that a receiver is fixed in an Earth Centered Earth Fixed (ECEF) reference frame during the interval \((t, t+1)\), the term \(\Delta \rho^s_r(t, t+1)\) depends upon the change in the geometric range due to satellite’s orbital motion and Earth’s rotation\(|\Delta \rho^s_r(t, t+1)|_{OR}\). However, it is also dependent on the variation of the solid Earth tide and ocean loading \(|\Delta \rho^s_r(t, t+1)|_{EIOI}\)[55], so that

\[
\Delta \rho^s_r(t, t+1) = |\Delta \rho^s_r(t, t+1)|_{OR} + |\Delta \rho^s_r(t, t+1)|_{EIOI}
\]

On the other hand, if we hypothesize that the receiver underwent a 3D displacement \(\Delta \xi_r(t, t+1)\) in an ECEF reference frame during the interval \((t, t+1)\), the term \(\Delta \rho^s_r(t, t+1)\) also includes the effect of \(\Delta \xi_r\) projected along the line-of-sight, which is observed over two consecutive epochs \((t \text{ and } t+1)\) if high-rate \((\geq 1 \text{ Hz})\) observations are utilized. Therefore, it is possible to write

\[
\Delta \rho^s_r(t, t+1) = [\Delta \rho^s_r(t, t+1)]_{OR} + [\Delta \rho^s_r(t, t+1)]_{EIOI} + e^s_r \cdot \Delta \xi_r(t, t+1)
\]

where \(e^s_r\) is the unit vector from the satellite to the receiver at epoch \(t\), and the symbol \(\cdot\) indicates the scalar product between the vectors \(e^s_r\) and \(\Delta \xi_r(t, t+1)\).

Here, it should be suddenly noted that the displacement \(\Delta \xi_r(t, t+1)\), if divided by the interval which is observed over two consecutive epochs \((t, t+1)\), is
equal to the (mean) velocity over the interval \((t, t + 1)\) itself. Therefore, the displacement \(\Delta \xi_r(t, t + 1)\), which is expressed in length units, is basically equivalent to a velocity, and it will be referred to as “velocity” in the following discussion. To summarize, it is possible to say that the GPS receiver is being used as a velocimeter.

The tropospheric term \(\Delta T^s_r(t, t + 1)\), represents the variation of the tropospheric delay \(T^s_r\) during the interval \((t, t + 1)\). This term is here modeled computing the Zenith Tropospheric Delay (ZTD) \((\text{i.e. }T^s_r(Z^s_r = 0))\) according to the Saastamoinen model [69], using the standard atmosphere as defined in Berg [10] and applying the mapping function included in Saastamoinen model, as follows

\[
T^s_r(Z^s_r(t)) = \frac{0.002277 \cos(Z^s_r(t)) \cdot \left[ P + \left(\frac{1255}{T} + 0.05\right) \cdot e - \tan^2(Z^s_r(t)) \right]}{\cos(Z^s_r(t))} \tag{4.5}
\]

where \(Z^s_r\) is the zenith angle of the satellite \((s)\) with respect to the receiver \((r)\), the atmospheric pressure \((P)\) and the partial water vapor pressure \((e)\) are given in millibars, the temperature \((T)\) is given in Kelvin degrees and the resulting tropospheric delay is obtained in meters.

Given the definition of the tropospheric delay variation, introducing equation 4.4 in equation 4.2 and simplifying the notation by omitting the epochs \((t, t + 1)\), yields to

\[
\alpha[\lambda\Delta \Phi^s_r]_{L1} + \beta[\lambda\Delta \Phi^s_r]_{L2} = \left(\Delta \rho^s_{r, OR} + \Delta \rho^s_{r, EIOI}\right) + \left(\Delta \delta t^s_r + \Delta \delta t^s_s\right) + T^s_r + \Delta p^s_r + \Delta m^s_r + \Delta \epsilon^s_r \tag{4.6}
\]

which can be rewritten in the form of the so defined variometric equation as follows

\[
\alpha[\lambda\Delta \Phi^s_r]_{L1} + \beta[\lambda\Delta \Phi^s_r]_{L2} = (e^s_r \cdot \Delta \xi_r + c(\Delta \delta t_r - \Delta \delta t^s)) + \left(\Delta \rho^s_{r, OR} - c \Delta \delta t^s + \Delta T^s_r\right) + \left(\Delta \rho^s_{r, EIOI} + \Delta \rho^s_s\right) + \Delta m^s_r + \Delta \epsilon^s_r \tag{4.7}
\]

where \(\alpha[\lambda\Delta \Phi^s_r]_{L1} + \beta[\lambda\Delta \Phi^s_r]_{L2}\) are the time single-difference ionosphere-free observations; \((e^s_r \cdot \Delta \xi_r + c\Delta \delta t_r)\) are terms containing the four unknowns parameters (the 3D velocity \(\Delta \xi_r\), and the receiver clock error variation \(\Delta \delta t_r\)); \((\Delta \rho^s_{r, OR} - c \Delta \delta t^s + \Delta T^s_r)\) is the largest part of the known term that can be computed on the basis of known orbits and clocks and for the chosen tropospheric model; \((\Delta \rho^s_{r, EIOI} + \Delta \rho^s_s)\) is an additional much smaller known term that can be computed with proper models for all of the considered effects; and \(\Delta m^s_r\) and \(\Delta \epsilon^s_r\)
4.1. The variometric approach and its estimation model

are the multipath and the noise term, as described previously.

Here, it is useful to note that the hypothesis of dual frequency observations free from cycle slips can be relaxed. In fact, as for the Instantaneous Positioning strategy [19], the losses of lock and cycle slips can be easily recognized during the analysis of the time series of the estimated 3D velocities $\Delta \xi_r$. The corresponding epochs can be rejected by setting a suitable threshold. An example of the effect of cycle slips will be given in section 4.3. A position-based modified real-time sidereal filtering, as proposed in [23] could be used to mitigate the multipath $\Delta m_s^r$; however, at present, this effect is neglected. Equation (4.7) represents the functional model of the least squares estimation problem. Well known is that low elevation observations are usually noisier, such that the observations are weighted by the squared cosine of the satellite zenith angle ($Z^s_r$) [26, p. 144], as follows

$$w = \cos^2(Z^s_r)$$  \hspace{1cm} (4.8)

The least squares estimation of the 3D velocity is based upon the entire set of variometric equations 4.7, which can be written for two generic consecutive epochs ($t, t+1$). The number of variometric equations obviously depends on the number of satellites which are common to the two epochs. At least four satellites are necessary to estimate the four unknown parameters (the 3D velocity $\Delta \xi_r$ and the receiver clock error variation $\Delta \delta t_r$) for each consecutive epoch couple.

4.1.1 Least squares estimation

To give a deeper explanation of how the least squares problem is dealt with in Variometric Approach for Displacement Analysis Stand-alone Engine (VADASE) algorithm let consider the simplified (i.e. linearized) functional model that relates the $u$ unknowns and the $n$ observations

$$y = A x$$  \hspace{1cm} (4.9)

where $y$ [$n \times 1$] is the observations vector, $A$ [$n \times u$] is the design matrix containing the coefficients for the unknowns, and $x$ [$u \times 1$] is the vector with the $u$ unknown parameters. Considering equation 4.9, a consistent solution with respect to the parameters reads as

$$x = A^{-1} y$$  \hspace{1cm} (4.10)

where observations should be independent in order for $A^{-1}$ to be non singular. Here, it is important to underline that, in order for the inverse matrix $A^{-1}$ to exist, matrix $A$ should be square. In case the number of observations is greater than the number of unknowns the problem becomes overdetermined and $n - u$ is referred to as the redundancy. Redundant problems are usually solved using least squares estimation technique.
4.1. The variometric approach and its estimation model

Generally, observations are affected by noise and uncertainty which are assumed to be Gaussian normally distributed with zero mean and to have a covariance matrix denoted as $\Sigma_\nu$ (e.g. $\nu \sim N(0, \Sigma_\nu)$) [42, p. 239]. Equation 4.9 yields to

$$y = A x + \nu$$

(4.11)

which corresponds to a general form of a Gauss-Markov model. Here, the functional model is expressed as

$$E[y] = A x, \quad E[\nu] = 0$$

(4.12)

and the stochastic model reads as

$$D[y] = \Sigma_y = \sigma_0^2 Q_y$$

(4.13)

where $E[\cdot]$ is the expectation operator and $D[\cdot]$ is the dispersion operator which describes the covariance matrix of the observations $\Sigma_y$. $\sigma_0^2$ is referred to as the a priori variance of unit weight and it is often assumed to be 1, $Q_y$ corresponds to the cofactor matrix. The inverse of the cofactor matrix is denoted as the weight matrix

$$W = Q^{-1} y$$

(4.14)

Since we assume that the observations are independent from one another, the weight matrix becomes diagonal (i.e. the terms expressing the correlation between the observations are equal to zero). The strategy to solve equation 4.11 is to constrain the sum of the squares of the residuals to be minimum

$$\nu^T W \nu = (y - A x)^T W (y - A x) = \text{minimum}$$

(4.15)

The vector of unknowns is estimated with

$$\hat{x} = (A^T W A)^{-1} A^T W y = N^{-1} y$$

(4.16)

where $N$ is denoted as the normal matrix. It useful to recall that any bias or outlier in the observations will deteriorate the models (i.e. mathematical and stochastic) and result into faulty parameters and faulty statistical values [42, p. 240]. The estimated values are used to retrieve the residuals

$$\hat{\nu} = y - A \hat{x}$$

(4.17)

which are then used to estimate the a posteriori variance of unit weight

$$\sigma_0^2 = \frac{\hat{\nu}^T W \hat{\nu}}{n - u}$$

(4.18)

A practical example of least squares estimation technique applied to VADASE
algorithm is further described. Assume that \( n \) satellites are viewed by the receiver both at epochs \( t \) and \( t + 1 \), the terms of equation 4.11 display as follows

\[
y_{n \times 1} = \begin{bmatrix}
\alpha[\lambda \Delta \Phi^1_r]_{L1} + \beta[\lambda \Delta \Phi^1_r]_{L2} \\
\alpha[\lambda \Delta \Phi^2_r]_{L1} + \beta[\lambda \Delta \Phi^2_r]_{L2} \\
... \\
\alpha[\lambda \Delta \Phi^i_r]_{L1} + \beta[\lambda \Delta \Phi^i_r]_{L2} \\
... \\
\alpha[\lambda \Delta \Phi^n_r]_{L1} + \beta[\lambda \Delta \Phi^n_r]_{L2}
\end{bmatrix}
\]

\[
x_{4 \times 1} = \begin{bmatrix}
\Delta X \\
\Delta Y \\
\Delta Z \\
\Delta \delta t_r
\end{bmatrix}
\]

\[
b_{n \times 1} = \begin{bmatrix}
[\Delta \rho^1_r]_{OR} + \Delta T^1_r - c \Delta \delta t^1 \\
[\Delta \rho^2_r]_{OR} + \Delta T^2_r - c \Delta \delta t^2 \\
... \\
[\Delta \rho^i_r]_{OR} + \Delta T^i_r - c \Delta \delta t^i \\
... \\
[\Delta \rho^n_r]_{OR} + \Delta T^n_r - c \Delta \delta t^n
\end{bmatrix}
\]

where, \( n \) is the number of observations and \( u = 4 \) is the number of unknowns. To be consistent with the exposition of the variometric algorithm given in 4.7, it is possible to introduce the vector \( b \) containing the so called known terms and to reformulate the general problem as

\[
y = Ax + b + \nu
\]  

(4.19)

The observations are assumed to be uncorrelated and are weighted by the squared cosine of the satellite zenith angle \( Z_s^r \)

\[
W_{n \times n} = \begin{bmatrix}
cos^2(Z_1^r) & 0 & \cdots & 0 \\
0 & cos^2(Z_2^r) & \cdots & 0 \\
... & ... & ... & ... \\
0 & 0 & \cdots & cos^2(Z_n^r)
\end{bmatrix}
\]

The least squares estimation of the unknowns is given as

\[
\hat{x} = (A^T W A)^{-1} A^T W (y - b)
\]  

(4.20)

### 4.2 Application of the variometric algorithm to a simulated example

In this section the outcomes of a first test designed to prove the functionality and the effectiveness of VADASE algorithm are presented. The introduced variomet-
ric approach was proven, by implementing a tuned software that was applied, initially, to a simulated example. The main goal of this test was to prove the potential of VADASE for estimating the velocities $\Delta \xi_r$ related to two consecutive measurements epochs $(t, t + 1)$ using GPS broadcast products (i.e. orbits and clocks).

Let begin considering the actual stream of carrier phase observations (file REALM0SE) collected at a rate of 1 Hz by the M0SE GPS permanent station (Rome, Italy) in the interval from 04:00:00 to 04:59:59, February 20, 2010. In order to introduce the effect of a known displacement into these observations, the simulation tool included in the Bernese software [26] was used. This tool, named GPSSIM, is able to simulate GPS observations that can be collected at a certain location under certain satellite scenarios (orbits, clocks, EOPs). In addition, the effects of range biases due to different error sources (e.g. tropospheric refraction, ionospheric refraction, satellite clock error, receiver clock error, ... ) can be added to the simulated observations.

At first, GPSSIM was used to generate the error-free carrier phase observations by fixing the position of M0SE to its known ITRF2005 coordinates and by using IGS precise products (orbits, clocks, EOPs) for the interval from 04:00:00 to 04:59:59, February 20, 2010, as a satellite scenario (file SIMUM0SE).

Then, error-free carrier phase observations were generated by referring to a position of M0SE that was shifted 1 cm to the East and to the North and 2 cm Up with respect to ITRF2005 coordinates. Again, the same satellite scenario and the same interval described above were used for generating the observations (file SIMUM0SE-SHFT). Hence, the differences (file DIFFM0SE equal to file SIMUM0SE-SHFT minus file SIMUM0SE) between the corresponding (i.e. same epoch, same satellite, same frequency) error-free carrier phase observations represent the error-free effect of the synthetically imposed (East, North, and Up) 3D displacements (1 cm, 1 cm and 2 cm, respectively) on the carrier phase observations themselves.

Finally, assuming that a 3D displacement from the ITRF2005 position to the shifted position occurred at epoch 900 (where epoch 0 is taken at 04:00:00, February 20, 2010) and that the displacement back to the original ITRF2005 position occurred at epoch 905, the differentiated values were summed from epoch 900 to epoch $905$ to actual carrier phase value (file REALM0SE) producing the new file REALM0SE-SHFT. At this point, REALM0SE-SHFT contained a stream of carrier phase observations that were impacted by the real noise and that included the effect of the imposed 3D displacement from epoch 900 to epoch 905. VADASE algorithm was tested on this simulated stream of data looking for the imposed 3D displacement which caused a 3D velocity $\Delta \xi_r$ from epoch 899
to epoch 900 and the 3D opposite velocity from epoch 904 to epoch 905, when the original ITRF2005 position was recovered.

The results are really encouraging: the East, North, and Up displacements of +1 cm, +1 cm and +2 cm at epoch 900 and of -1 cm, -1 cm and -2 cm at epoch 905 are clearly visible in the time series of the estimated 3D velocities $\Delta \xi_r$ (figure 4.1).

![Figure 4.1: M0SE – estimated 3D velocities using GPS broadcast products (orbits and clocks) available in real-time (BRD solution) and the best quality products (orbits and clocks) supplied by IGS a posteriori (PRE solution) in the 1 hour interval 04:00:00–04:59:59, February 20, 2010 GPS time](image)

Displacements were estimated with an accuracy of $1 \div 2$ mm in both the horizontal and the up directions by using the GPS broadcast products (orbits and clocks) available in real time (BRD solution) and the best quality products (orbits and clocks) supplied by IGS a posteriori (PRE solution) (table 4.1). For the Up component, it is worth noting how the velocities initially display a bias of approximately 1 cm. This effect lasts for less than 500 epochs and is most probably related to some issues in the very first implementation of the variometric model (i.e. at this initial stage, only the geometric part of the model was implemented and all the disturbing effects were still neglected). As a matter of fact, however, this small trend does not affect the algorithm’s effectiveness.
4.3. Cycle slips impact on the estimated velocities

Table 4.1: M0SE – comparisons between imposed displacements and estimated 3D velocities using GPS broadcast products (orbits and clocks) available in real-time (BRD solution) and the best quality products (orbits and clocks) supplied by IGS a posteriori (PRE solution) at epochs 899-900 and 904-905 of the 1 hour interval 04:00:00–04:59:59, February 20, 2010. GPS time

<table>
<thead>
<tr>
<th></th>
<th>899-900</th>
<th></th>
<th>904-905</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected</td>
<td>( \Delta E ) 0.010</td>
<td>( \Delta N ) 0.010</td>
<td>( \Delta Up ) 0.020</td>
</tr>
<tr>
<td>Estimated PRE</td>
<td>( \Delta E ) 0.010</td>
<td>( \Delta N ) 0.006</td>
<td>( \Delta Up ) 0.018</td>
</tr>
<tr>
<td>Estimated BRD</td>
<td>( \Delta E ) 0.011</td>
<td>( \Delta N ) 0.007</td>
<td>( \Delta Up ) 0.020</td>
</tr>
</tbody>
</table>

In addition, the global agreement between the BRD and the PRE solutions in term of the standard deviation of 3D velocities differences was evaluated. The agreement was within 1 mm for the horizontal coordinates and 2 mm for the height [25].

4.3 Cycle slips impact on the estimated velocities

One of the principal advantages of the variometric algorithm lies in the fact that there is no need to solve for the integer initial ambiguities which affect the carrier phase observations. In fact, as shown in equation 4.2, with the hypothesis that no cycle slips occur the single-difference between epochs \( t \) and \( t + 1 \) cancels out the ambiguity term.

On the other hand, in case a cycle slip occurs, its effect would be easily recognized as an outlier in the analysis of the time series of estimated 3D velocities \( \Delta \xi \). In addition, proper data snooping techniques could be used to detect the observations affected by the cycle slip and to exclude them from being used in the epoch solution.

In this section, the effect of cycle slips on the variometric algorithm will be reported and the results of VADASE algorithm over a simulated example will be discussed. To simulate GPS observations useful to the present goal it was used the simulation tool \( \text{GPSSIM} \), which is a tool of the Bernese software, already described in section 4.2.

At first, \( \text{GPSSIM} \) was used to generate carrier phase observations at 1 Hz rate fixing the position of M0SE to its known ITRF2005 coordinates using GPS...
4.3. Cycle slips impact on the estimated velocities

broadcast products (orbits, clocks) for the interval from 12:15:00 to 12:45:00, August 1, 2011. All the effects that could bias the signal propagation with respect to the “pure” geometric path were excluded. The observations noise for both carrier phases \( L1 \) and \( L2 \) was set to the default value suggested by Bernese software [26] (i.e. \( 10^{-3} \text{ m} \)).

The simulated observations were stored in a Receiver INdependent EXchange format (RINEX) file and processed using VADASE software. Here, since the atmospheric biases do not affect the observations, the variometric model has been modified to be compliant with the conditions used in the simulation. The simplified variometric equation reads as follows

\[
\lambda \Delta \phi_r^{s, r} = (e_r \bullet \Delta \xi_r + c \Delta \delta t_r) + (\Delta \rho_{\text{OR}}^s - c \Delta \delta t^s)_{BRD} + \Delta \epsilon_r^s \quad (4.21)
\]

where the effects due to ionosphere, troposphere, phase center variation, relativity, phase wind-up and multipath have been neglected. The known term \((\Delta \rho_{\text{OR}}^s - c \Delta \delta t^s)_{BRD}\) was computed using GPS broadcast products. Equation 4.21 represents the simplified functional model of the least squares estimation problem. As regards the stochastic model, all the observations have been weighted equally.

The least squares estimation of the 3D velocities is based upon the entire set of the variometric equation 4.21 that can be written (separately for \( L1 \) and \( L2 \)) for two generic consecutive epochs \( t \) and \( t + 1 \). It is worth noting that the simplified functional model allows to double the overall number of variometric equations with respect to the complete model.

The results in terms of estimated receiver velocities are shown in figure 4.2. The red dots depicts the velocity values for the East, North and Up components whereas the number of observations used in each consecutive epoch \( t, t + 1 \) is shown in green dots. At epoch 130894.0, a change in the satellite constellation configuration brings down the observations number from 18 (i.e. 9 satellites) to 16 (i.e. 8 satellites).

At this point, the simulation was repeated with the same conditions described above but allowing GPSSIM to introduce cycle slips in the carrier phase observations. 4 cycle slips were introduced randomly in the observations [26, p. 352]. In order to verify the effects of the smallest possible jump in the carrier phase observations, the maximum size of the slips was set to 1 cycle. Such conditions were chosen in order to test the variometric algorithm under the most challenging situation for which a cycle slip has to be detected: there is no loss of lock of the signal and the size of the slip is only 1 cycle. Most likely, in fact, cycle slips are worth hundreds of cycles and are accompanied by a loss of lock of the signal, thus being more easily recognizable.
Figure 4.2: M0SE – estimated 3D velocities using GPS broadcast products (orbits and clocks) in the 30 min interval 12:15:00-12:45:00, August 1, 2011 GPS time. The number of observations used to solve for the velocity is reported in green.

The new simulated observations were processed again by VADASE. The results are shown in figure 4.3. The time series of the estimated 3D velocities displays 4 outliers (i.e. at epochs 130649.0, 130906.0, 131472.0 and 131540.0) that are caused by the presence of cycle slips in the observations. To highlight the impact of the slips in the variometric algorithm, an example of the observations that are used to build the variometric combination relative to a given epoch couple (i.e. epochs 130648.0 and 130649.0) are shown in table 4.2.

The “Known term” column contains the values of $\Delta \rho_s - c \Delta \delta t$. This “Known term” is computed using GPS broadcast products. The column “$\Delta L$” contains the observations difference between the two epochs (i.e. the variometric observation). Finally, column “$\Delta L$ - Known term” reports the difference between the two quantities.

At this point, it is worth focusing on the latter column to make a further, important, consideration. The difference between the observation and the computation of the known term should return very similar values for each of the considered satellites. In addition, these values should normally be scattered, possibly with a low dispersion, around a common amount that corresponds to the
4.3. Cycle slips impact on the estimated velocities

<table>
<thead>
<tr>
<th>Id</th>
<th>Known term</th>
<th>$L_{1,2}$</th>
<th>$\Delta L$</th>
<th>$\Delta L - \text{Known term}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G02</td>
<td>−420.366</td>
<td>L1W</td>
<td>−420.364</td>
<td>0.002</td>
</tr>
<tr>
<td>G02</td>
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<td>L2W</td>
<td>−420.367</td>
<td>−0.001</td>
</tr>
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<td>−5.435</td>
<td>L1W</td>
<td>−5.436</td>
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</tr>
<tr>
<td>G04</td>
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<td>L2W</td>
<td>−5.435</td>
<td>0.001</td>
</tr>
<tr>
<td>G09</td>
<td>390.547</td>
<td>L1W</td>
<td>390.545</td>
<td>−0.003</td>
</tr>
<tr>
<td>G09</td>
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<td>L2W</td>
<td>390.547</td>
<td>−0.001</td>
</tr>
<tr>
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<td>−0.001</td>
</tr>
<tr>
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<td>13.689</td>
<td>L2W</td>
<td>13.692</td>
<td>0.002</td>
</tr>
<tr>
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<td>L1W</td>
<td>−233.167</td>
<td>−0.189</td>
</tr>
<tr>
<td>G14</td>
<td>−232.980</td>
<td>L2W</td>
<td>−232.980</td>
<td>−0.000</td>
</tr>
<tr>
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<td>L1W</td>
<td>685.810</td>
<td>−0.000</td>
</tr>
<tr>
<td>G15</td>
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<td>685.812</td>
<td>0.001</td>
</tr>
<tr>
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<td>L1W</td>
<td>−362.449</td>
<td>0.000</td>
</tr>
<tr>
<td>G25</td>
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<td>L2W</td>
<td>−362.448</td>
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</tr>
<tr>
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<td>L1W</td>
<td>374.908</td>
<td>−0.001</td>
</tr>
<tr>
<td>G27</td>
<td>374.909</td>
<td>L2W</td>
<td>374.910</td>
<td>0.001</td>
</tr>
<tr>
<td>G29</td>
<td>−672.741</td>
<td>L1W</td>
<td>−672.742</td>
<td>−0.001</td>
</tr>
<tr>
<td>G29</td>
<td>−672.741</td>
<td>L2W</td>
<td>−672.740</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 4.2: Observations used to solve epochs difference
4.3. Cycle slips impact on the estimated velocities

Figure 4.3: M0SE – estimated 3D velocities using GPS broadcast products (orbits and clocks) in the 30 min interval 12:15:00-12:45:00, August 1, 2011 GPS time. The number of observations used to solve for the velocity is reported in green variation of the receiver clock error between the considered epochs. In this simulated example, since the data were generated without considering the receiver clock offset, the values of the latter column are distributed around zero. On the other hand, in case real data were processed, the variation of the receiver clock offset, whose order of magnitude strongly depends on the receiver hardware, would have been thereby included.

Now, by looking at the values displayed in this last column it appears clear that a proper data snooping technique can be used to identify the observation(s) which is (are) affected by a cycle slip (i.e. $L_1$ carrier phase observation for satellite G14) and to exclude it (them) before the solution is estimated for epochs $t$ and $t+1$.

Table 4.3 shows the characteristics of the cycle slips introduced by Bernese software in the simulated observations. For all the cases the presence of the slip is highlighted. Moreover, neglecting the receiver clock offset in the simulation, the size of the slips is retrieved accurately by looking at the term containing the difference between observations and known term differences for epochs $t$ and $t+1$. 

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### 4.3. Cycle slips impact on the estimated velocities

<table>
<thead>
<tr>
<th>Bernese simulated slips</th>
<th>VADASE retrieved slips</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq</td>
</tr>
<tr>
<td>1</td>
<td>450</td>
</tr>
<tr>
<td>2</td>
<td>707</td>
</tr>
<tr>
<td>2</td>
<td>1273</td>
</tr>
<tr>
<td>1</td>
<td>1341</td>
</tr>
</tbody>
</table>

Table 4.3: Cycle slips values retrieved by VADASE
4.3. Cycle slips impact on the estimated velocities

4.3.1 Addressing cycle slips with a proper data snooping technique

Up to now, it was shown that possible cycle slips in the observations are easily recognized as outliers and can be removed from the time series of estimated 3D velocities $\Delta \xi_r$ (see figure 4.3). However, this exclusion generates a loss of information that might be crucial, especially in case the slip occurs within a time interval when the receiver motion should be dynamically characterized.

To address this issue, it has been shown that the analysis of the term resulting from the subtraction of the observations difference and the known term difference between the generic epoch couple $t$ and $t+1$ allows to evidence which observation(s) is (are) spoiled by the cycle slips (see table 4.2). This attestation is of high significance since it opens up the possibility of adopting a proper DS technique to detect and eliminate the observations affected by the slips, thus avoiding that the velocity estimation results as an outlier and is thereby removed from the velocities time series.

To this aim, a suited DS technique has been implemented in VADASE software and tested with the simulated data formerly generated by GPSSIM tool. The implemented technique is rather simple and it is expressed in terms of the following pseudocode:

1. form the variometric terms (i.e. $\Delta L$, “Known term” and $L_K$; where $L_K = \Delta L - “Known term”) using all the $n$ observations available for the generic couple of consecutive epochs $t$ and $t+1$

2. compute the mean value ($\mu$) of $L_K$

3. for each element $l_k_i$ in $L_K$, evaluate the absolute value $|l_k_i - \mu|$

4. compute the maximum ($max_i$) of the $n$ elements $|l_k_i - \mu|$

5. if $max_i$ is bigger than $\lambda/2$ (being $\lambda$ the observation wavelength), exclude observation $i$ from $L_K$ and go to step 2; else, go to step 6

6. proceed to least squares estimation of the receiver velocity

Following the implementation of the described DS technique in VADASE software, the synthetic data containing cycle slips were processed again. The results are shown in figure 4.4. Although the implemented technique is rather simple, it shows to effectively address the cycle slips and to successfully exclude the outliers from the observations used to solve for the generic epoch couple. This behavior is highlighted by the green dots displayed in the plot. In fact they show that
4.3. Cycle slips impact on the estimated velocities

the number of observations used in the velocity solution lowers by one unit in correspondence of the epochs when cycle slips impact the data.

Overall, the solutions are continuous and there is no loss of information.

Figure 4.4: M0SE – estimated 3D velocities using GPS broadcast products (orbits and clocks) and applying the described Data snooping technique, in the 30 min interval 12:15:00-12:45:00, August 1, 2011 GPS time. The number of observations used to solve for the velocity is reported in green
Chapter 5

Spirent GNSS simulator

A Spirent simulator provides comprehensive facilities for development and production testing of Satellite Navigation equipment and for integration studies. The simulator reproduces the environment of a navigation receiver installed on a dynamic platform. Radio Frequency (RF) signals are simulated exhibiting the effects of high-dynamic host vehicle motion, navigation satellite motion and ionospheric and tropospheric effects [74].

The simulator consists of two primary elements:

- the simulation hardware
- the SimGEN software

The simulation hardware

The GSS7790 Multi-Output Modernized-GNSS Signal Simulator is a totally unique product. It has been specifically developed by Spirent to provide the core element in GPS test applications that require independent access to each simulated satellite signal at RF. The GSS7790 is a full constellation simulator, offering total user control over satellite orbital definition. It accurately models the resulting satellite motion with respect to the user-specified simulation location, date and time. The hardware consists of one or more multi-channel RF signal generators. A signal generator provides signals that stimulate the receiver or sensor under test or evaluation. This can be considered as a “pseudorange to RF” converter. Each channel represents a satellite signal at a single carrier frequency. A com-
plete description of the features of the hardware used within this work can be found in [57].

SimGEN

SimGEN allows the specification, development and execution of simulations. The simulation takes the form of a scenario, which uses a set of description, or source files, each of which has its own specific editor. The main parameters that define a scenario are the following:

• GPS/GLONASS/Galileo constellation characteristics to be simulated
• tropospheric and ionospheric conditions
• receiver motion
• position of the receiving antenna on the vehicle, gain pattern . . .

The main task of SimGEN is concerned with allowing the user to define these parameters and saving them as complete scenarios.

During a simulation, SimGEN runs models that act on the source file information and it calculates the pseudorange between the simulated visible satellites and the simulated vehicle in which the receiver is installed. SimGEN provides information required by the signal generator, such as navigation data, signal arrival angle and power level. Comprehensive facilities allow manipulation of the simulated satellite orbits and the navigation data describing them. Spirent permits to simulate a wide range of normal and abnormal situations. Further information can be found in [74].

The simulation hardware available at DLR consists of two GSS7790 core modules for GPS and Galileo satellite systems (Fig. 5.1).

In the framework of the cooperation undertaken between DLR (Navigation and Communication Institute) and “Sapienza” University of Rome (Area di Geodesia e Geomatica), Spirent simulator has been used to develop and assess VADASE algorithm. Several scenarios have been generated and the related data were collected in the form of RINEX files.

In details, starting from the simplest possible condition (i.e. signals traveling without any disturbances along the geometric path from the generic satellite to the receiver), each component perturbing signal travel path has been simulated separately in order to determine its influence over waveforms estimation. Then, RINEX data have been processed using VADASE, aiming to retrieve the movements imposed to the receiver and to check the effectiveness of the variometric model described in equation 4.7.
The achieved results allowed the algorithm to be corrected, enriched and improved where needed. Moreover, the impact of some relativistic effects on the solutions was analyzed in details.

**Receiver hardware**

Two dual frequency geodetic receivers have been alternatively used to record and acquire the simulated data:

- Javad Delta G3T receiver (figure 5.2 a)

- Septentrio PolarRx3 receiver (figure 5.2 b)

They both have multi-constellation tracking capabilities (i.e. Javad Delta receiver can acquire GPS/GLONASS/Galileo signals whereas Septentrio PolarRx3 receiver can acquire GPS/GLONASS signals) and the both support high acquisition rate (i.e. Javad Delta receiver can acquire data at a maximum rate of 50 Hz whereas Septentrio PolarRx3 reaches a maximum rate of 20 Hz).
5.1 VADASE solutions for a simple simulation: geometric ranges

VADASE velocities and waveforms estimation is described in this section by showing the results of the first scenario simulated with Spirent. This consists on the receiver remaining fixed in its initial position (set by the user) and acquiring GPS and Galileo signals generated by the simulator.

Both constellations have been simulated using their standard characteristics, which are internally implemented in Spirent. Any other effect disturbing the signal propagation along the path from the satellite to the receiver has been excluded from the scenario. The complete variometric model (equation 4.7) was adapted in order to be compliant with the characteristics of the present scenario (i.e. scenario main parameters are displayed in table 5.1). The tuned model reads as follows

\[
[l_{12}^s \Phi_r] = (e^s_r \cdot \Delta \xi_r + c \Delta \delta t_r) + [\Delta \rho_{12}^s]_{OR} + \Delta \epsilon_r^s
\]  

(5.1)
where the terms representing the effects of satellite clock offset, ionosphere, troposphere, Earth tides, ocean loading, phase center variation, relativity and phase wind-up were neglected because they were not included in the simulated data. Regarding the stochastic model, since the signals were not disturbed by the presence of the atmosphere, all the observations have been weighted equally.

The least squares estimation of the 3D velocity was based upon the entire set of the variometric equation 5.1, which can be written (separately for L1 and L2) for each satellite in common to two generic consecutive epochs \((t, t + 1)\). Javad Delta receiver was used for this first test. The simulated data were acquired at 20 Hz rate and converted in RINEX files (observation and navigation files, version 3.00).

**GPS solutions**

*VADASE* velocities solutions for GPS constellation are shown in figure 5.3.

![Image](image_url)

**Figure 5.3:** Receiver velocities estimation (in red) using GPS constellation for 10 minutes interval. Green dots show the number of observations used in the least squares solution. Data are obtained from Spirent simulator using the scenario described in table 5.1.
5.1. VADASE solutions for a simple simulation: geometric ranges

On the left-hand y axis the displacements between two consecutive epochs \((t, t + 1)\) are displayed, expressed in length units. On the right-hand y axis, the number of observations used to solve the least squares problem is shown in green (it is handful to recall that, for each satellite, L1 and L2 carrier phase observations are used separately). The accuracy of the estimated displacements, which can be thought of as an index of the minimum displacement detectable by VADASE, is at 1 millimeter level for the horizontal components and at 3 millimeters level for the vertical one, as reported in table 5.2.

<table>
<thead>
<tr>
<th>GPS</th>
<th>Statistics on estimated velocities</th>
</tr>
</thead>
<tbody>
<tr>
<td>12000 epochs</td>
<td>ΔClock</td>
</tr>
<tr>
<td>average [m]</td>
<td>-0.909</td>
</tr>
<tr>
<td>std dev [m]</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Table 5.2: Scenario 1: statistical characterization of the estimated velocities with GPS constellation

At this point, provided that the time series of the receiver displacements were estimated, it is possible to describe the dynamic movements (referred to as waveforms) of the receiver during the time of interest. To this aim, assuming that continuous data have been acquired, the time series of the estimated 3D velocities are integrated (for the 3 different components) in order to achieve the receiver waveforms.

Well known is that (discrete) integration is very sensitive to estimation biases. Here, the estimated velocities might be biased because of two main effects:

- errors in the estimation model
- residuals due to defects of the applied model

These biases accumulate over time and show their signature as a trend in the integrated displacements. The receiver waveforms obtained for the three components by discrete integration of the estimated 3D velocities are exposed in figure 5.4.

None of the three components shows a movement with respect to its proper initial position. Overall, the 3D receiver displacement at the end of the analyzed interval results in 0.003 m. This outcome, though being different from zero because of the observations noise, fulfills the expectations. In fact, it duly reflects the conditions adopted to simulate the data.
5.1. VADASE solutions for a simple simulation: geometric ranges

Figure 5.4: Receiver waveforms (integrated velocities) (in red) using GPS constellation for 10 minutes interval. Green dots show the number of observations used in the least squares solution. Data are obtained from Spirent simulator using the scenario described in table 5.1

Galileo solutions

The same analysis was carried out using the variometric algorithm with the signals broadcast from Galileo constellation.

By looking at figure 5.5, which shows the velocities solution obtained with VADASE, it is important to underline the following:

- the level of noise in the solution decreases (especially for the East and Up components) when the number of observations used in the least squares estimation increases from 12 to 14. This behavior is confirmed by the improvement in the Position Dilution of Precision (PDOP)\(^1\) index that improves from 2.09 to 1.30 consequently to the increment in satellite number. A similar effect was not immediately notable in the solution obtained employing GPS constellation (figure 5.3). In that case, the PDOP index measured

\(^{1}\)Dilution of Precision (DOP) indexes measure the quality of the positioning solution that can be achieved according to the geometric configuration of the satellites in view. The lower the indexes, the better the achievable accuracy
5.1. VADASE solutions for a simple simulation: geometric ranges

Figure 5.5: Receiver velocities estimation (in red) using Galileo constellation for 10 minutes interval. Green dots show the number of observations used in the least squares solution. Data are obtained from Spirent simulator using the scenario described in table 5.1

1.55 with 16 observations (8 satellites) and improved to 1.30 with 18 observations (9 satellites). Overall, it should be underlined that these changes are strongly dependent on the geometry of the satellite constellation.

- at epoch 174625.00 [sec of week], the estimated velocities display one outlier.

The estimated velocities (table 5.3) are as accurate as those obtained using GPS constellation.

At this point, the time series of the estimated 3D velocities are integrated in order to obtain the receiver movements. The results are shown in figure 5.6.

In this case, the integrated velocities display a trend in all the components. The presence of the outlier is clearly recognizable as a jump in the receiver waveforms. Overall, excluding the outlier from the time series, the 3D displacement at the end of the considered interval reaches 0.038 m.

Here, it is worth remarking that drifts in Galileo integrated velocities reflect the presence of a bias in the solutions estimated by the variometric algorithm. This could be due to possible errors that arise in the computation of satellites
5.1. VADASE solutions for a simple simulation: geometric ranges

<table>
<thead>
<tr>
<th>Galileo Statistics on estimated velocities</th>
</tr>
</thead>
<tbody>
<tr>
<td>12000 epochs</td>
</tr>
<tr>
<td>average</td>
</tr>
<tr>
<td>std dev</td>
</tr>
</tbody>
</table>

Table 5.3: Scenario 1: statistical characterization of the estimated velocities with Galileo constellation

Figure 5.6: Receiver waveforms (integrated velocities) (in red) using Galileo constellation for 10 minutes interval. Green dots show the number of observations used in the least squares solution. Data are obtained from Spirent simulator using the scenario described in table 5.1

coordinates. Although the mishandling of the differences in the time reference systems between GPS and Galileo constellations is most likely the reason of the displayed effect, further investigations have already been planned in order to have a deeper insight into this issue.
5.1. VADASE solutions for a simple simulation: geometric ranges

Combined GPS and Galileo solutions

As it was shown in previous subsections, the variometric algorithm implemented in VADASE software can be applied using signals from different constellations. In addition, GNSS constellations interoperability is a key factor in order to improve the positioning accuracy and to foster the operational continuity and the solution robustness of the algorithm. In this framework, VADASE software has been consistently implemented to achieve solutions profiting from the combination of multiple signals from different constellations.

Here, for the data generated with the previously described scenario, the receiver velocities were estimated stacking GPS and Galileo observations for L1, L2 and L5 carrier phases. The solutions were referred to the GPS time system. To this aim, it was necessary to account for the GPS to Galileo Time Offset (GGTO) (i.e. a broader discussion on the real-time computation of GGTO is given in [32, p. 51]). The results are shown in figure 5.7. The accuracy of the estimated displacements, is approximately 2 millimeters both for the horizontal and the vertical components, as it is shown in table 5.4. The estimated velocity solution for epoch 174625.0 results as an outlier, which is most notable for the East component.

<table>
<thead>
<tr>
<th>GPS + Galileo</th>
<th>Statistics on estimated velocities</th>
</tr>
</thead>
<tbody>
<tr>
<td>12000 epochs</td>
<td>∆Clock</td>
</tr>
<tr>
<td>average [m]</td>
<td>-0.909</td>
</tr>
<tr>
<td>std dev [m]</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Table 5.4: Scenario 1: statistical characterization of the estimated velocities with GPS and Galileo constellations

The integration of the estimated velocities for the analyzed period yields to receiver waveforms, which are shown in figure 5.8.

Here, the combined solution displays a slight drift. This can be explained as the result of the combination of the plain waveforms obtained with GPS constellation and the drifted waveforms achieved with Galileo.

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2Simulated examples were shown for GPS and Galileo but, from a theoretical standpoint, there are no limitations to extend the algorithm towards any other (even future) constellations
5.1. VADASE solutions for a simple simulation: geometric ranges

Figure 5.7: Receiver velocities estimation (in red) using GPS and Galileo constellations for 10 minutes interval. Green dots show the number of observations used in the least squares solution. Data are obtained from Spirent simulator using the scenario described in table 5.1
5.1. VADASE solutions for a simple simulation: geometric ranges

Figure 5.8: Receiver waveforms (integrated velocities) (in red) using GPS and Galileo constellations for 10 minutes interval. Green dots show the number of observations used in the least squares solution. Data are obtained from Spirent simulator using the scenario described in table 5.1
5.1. VADASE solutions for a simple simulation: geometric ranges

Remarks and future improvements

VADASE results have been presented and explained in details while analyzing the first simulation scenario. Initially, 3D velocities were estimated using the variometric equations described in 4.7. Then, to reconstruct the receiver dynamic movements in a global reference frame, the time series of the velocities were integrated.

The set of parameters chosen for this first simulation scenario allowed to investigate how the variometric algorithm is capable to model the geometric term \([\Delta \rho_s^r]_{OR}\) described in equation 5.1 using broadcast orbit provided in the satellite message. Satellite signals generated from Spirent were acquired and reformed in RINEX 3.0 files using Javad Delta receiver and the related conversion software.

At first, for each constellation (i.e. GPS and Galileo), separate solutions were obtained using L1 and L2 carrier frequency observations. Then, a combined solution was achieved stacking the observations of both constellations. The standard deviation of the estimated velocities for the three cases are reported in table 5.5.

<table>
<thead>
<tr>
<th></th>
<th>12000 epochs</th>
<th>∆Clock [m]</th>
<th>∆East [m]</th>
<th>∆North [m]</th>
<th>∆Up [m]</th>
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<tr>
<td>GPS</td>
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<td>0.001</td>
<td>0.001</td>
<td>0.003</td>
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<tr>
<td>Gal</td>
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<td>0.001</td>
<td>0.001</td>
<td>0.003</td>
<td></td>
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<tr>
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<td>0.002</td>
<td>0.001</td>
<td>0.002</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.5: Statistical results summary

GPS and Galileo solutions reach an accuracy of about 1 millimeter in the horizontal components and about 3 millimeters in the vertical one. Stacking the observations of both satellite systems, Up and North components slightly increase their level of accuracy, while East accuracy drops marginally.

Considering receiver movements obtained at the end of the 10 minutes interval chosen for the analysis, the obtained results differ according to the used constellation:

- 10 minutes processing of GPS observations yields to a 3D receiver displacement of 0.003 m. This small amount, most probably due to receiver noise, is in a strong agreement with simulation conditions decided for this scenario

- Galileo results display a trend in all three waveform components. This condition reflects the possible presence of a bias in the estimated velocities. The final amount of the 3D receiver displacement is 0.038 m
5.1. VADASE solutions for a simple simulation: geometric ranges

- the drift affecting Galileo solutions remains still present, though less marked thanks to the contribution of GPS constellation, in the combined solution

As a final remark, it is important to underline that the difference displayed in the results attaining the two constellations was not expected and needs still to be investigated in more details. A possible answer can be searched for in the small differences between GPS and Galileo coordinate and time reference systems\(^3\).

5.1.1 Geometry term assessment

Up to now, VADASE proved to compute effectively the time difference (i.e. between epoch \(t\) and \(t + 1\)) of the geometric range term \([\Delta \rho_{s}^{r}(t, t + 1)]_{OR}\), which depends on satellites orbital motion and Earth’s rotation. Broadcast ephemerides, provided as a part of simulated messages, were used to compute satellites positions employing the standard algorithm described in table 2.3. In details, to obtain the epoch time \(t\) at which satellites coordinates should be computed (i.e. when the signal was sent by the satellite), it is crucial to take into consideration the receiver clock error (i.e. \(\delta t_r\)) with respect to the time reference system (i.e. GPS time is assumed as reference).

Here, it follows a brief explanation of the iterative algorithm that is implemented in the variometric algorithm to compute satellites’ coordinates in an ECEF reference frame for a generic epoch \(t\).

Let start recalling the pseudorange observation equation from a generic satellite \(s\) to a receiver \(r\), as introduced in [42, p.105]

\[
P_s^r = c[t_r(t_{rec}) - t^s(t_{sat})] = c \Delta t + c(\delta t_r - \delta t^s) = \rho + c(\delta t_r - \delta t^s)
\]

where \(t_r(t_{rec})\) and \(t^s(t_{sat})\) are the time stamp read at the moment the signal reaches the receiver and leaves the satellite, respectively; \(\Delta t\) represents the true signal transmission time; \(\delta t_r\) and \(\delta t^s\) are the receiver and the satellite clock errors with respect to reference time system; \(\rho = c \Delta t\) is the geometric distance between the satellite and the receiver.

\[
\rho = \sqrt{(X^s - X_r)^2 + (Y^s - Y_r)^2 + (Z^s - Z_r)^2}
\]

and \(c\) is the speed of light.

\(^3\)Implementation issues are also under investigation. However, since the algorithm to determine satellite coordinates using the broadcast ephemerides is the same for both constellations, misusing in either Galileo time or coordinates reference systems are more likely the cause of these drifts.
5.1. VADASE solutions for a simple simulation: geometric ranges

Provided that satellites’ coordinates \((X^s, Y^s, Z^s)\) and clock error \((\delta t^s)\) can be computed at any given epoch by using ephemerides, it results clear that, if the user position \((X_r, Y_r, Z_r)\) is known, equation 5.2 can be solved for the receiver clock offset \((\delta t_r)\). Hence, at generic observation epoch \(t_r(t_{rec})\), any satellite which is in view provides a different solution for \(\delta t_r\). After testing different possibilities, it has been decided to use the arithmetic mean of satellites contributes to obtain the receiver clock offset for the given epoch.

Now, let define \(t_r\) as the time stamp read from the receiver clock when the signal is received. The first step of the algorithm consists on computing an initial value for satellites’ position which will be used to start the iterative procedure. Hence, formulas in table 2.3 are applied using \(t = (t_r - stt_0)\), where \(stt_0 = 0.075[s]\) is a reference initial value that can be adopted for signal transmission time from the generic satellite to the receiver.

To account for Earth’s rotation effect during the signal transmission time (also known as Sagnac effect, more details in appendix A), coordinates are further rotated by a quantity dependent on the signal transmission time. At this point, a first computation of satellites coordinates have been obtained and the iterative procedure can start. The generic iteration \(i\) consists on the following steps:

1. for each satellite in view, compute \(stt_i^s\)
2. solve pseudorange equation 5.2 for \(\delta t_{r,i}\)
3. for each satellite \(s\), compute new coordinates at epoch \(t^{s,i} = t_r - stt_i^s - \delta t_{r,i}\)
4. rotate coordinates to account for Earth’s rotation
5. compare satellites coordinates at iterations \(i\) and \(i - 1\)

The algorithm stops when coordinates difference between iterations \(i\) and \(i - 1\) is lower than a defined threshold (i.e. 1 cm). Generally, it takes up to three iterations for the algorithm to converge.

The described procedure, implemented in VADASE variometric algorithm, yields the good results in terms of estimated 3D receiver displacements, as reported in figure 5.4. Nonetheless, it is necessary to remark that this outcome corresponds to an ideal condition in which both satellites and receiver coordinates are known with the highest possible accuracy (i.e. due to simulation: receiver coordinates are fixed by the user, broadcast ephemerides allow to compute the satellite coordinates without uncertainties).

5.1.2 Sensitivity analysis on receiver position

To extend the assessment of the geometry term, it might be useful to consider how an uncertainty on receiver position (which is always the case in real field situa-
5.1. VADASE solutions for a simple simulation: geometric ranges

(omissions) would affect the variometric algorithm. Therefore, the same data simulated using parameters in table 5.1 were processed again with the variometric algorithm introducing a shift in receiver coordinates. Practically speaking, VADASE used a receiver position altered in each component by a certain amount with respect to the position used for data simulation. Two different conditions were tested:

- 1 meter error in each receiver component \((X_r, Y_r, Z_r)\), corresponding to a 3D position error of \(\sim 1.73\) meters
- 10 meters error in each receiver component \((X_r, Y_r, Z_r)\), corresponding to a 3D position error of \(\sim 17.32\) meters

Figure 5.9 shows the outcomes of this sensitivity analysis in terms of 3D receiver waveforms.

![Receiver waveforms](image)

Figure 5.9: Receiver waveforms (integrated velocities) using GPS constellation for 10 minutes interval. Green dots show the number of observations used in the least squares solution. Data are obtained from Spirent simulator using the scenario described in table 5.1.

For each component, the left-hand y axis displays results coming from the integration of the estimated velocities. Displacements corresponding to the “correct” receiver position are reported in red, whereas orange and blue are used to
5.1. VADASE solutions for a simple simulation: geometric ranges

depict the effects of the 1 and 10 meters error, respectively. On the right-hand y axis the number of the observations is shown using green.

Table 5.6 summarizes the 3D receiver displacement achieved by the variometric algorithm in 10 minutes (20 Hz acquisition rate) for the three different situations.

<table>
<thead>
<tr>
<th>Receiver position</th>
<th>3D receiver displacement $[m]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>“correct”</td>
<td>0.003</td>
</tr>
<tr>
<td>1 m error $(X_r, Y_r, Z_r)$</td>
<td>0.100</td>
</tr>
<tr>
<td>10 m error $(X_r, Y_r, Z_r)$</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 5.6: Sensitivity analysis on receiver initial coordinates: final 3D receiver displacement

In case the “correct” position is used, the algorithm returns a 3D receiver displacement of 3 millimeters in 12000 epochs. This small value, obtained integrating estimated velocities, shows the effects of receiver and observations noise. An uncertainty level of $\sim 1.7$ meters in the receiver position (which can be the case if pseudoranges observations are used to achieve the initial receiver coordinates) results in a distorted estimation of the 3D displacement, which reaches 10 centimeters. For the last case, if the receiver coordinates are uncertain at the level of $\sim 17$ meters, a 3D displacement of 1 meter is retrieved.

Overall, these results yield a strong proof of how a bias introduced in one of the terms of the variometric model (in this case it was the receiver coordinates) reflects as a drift in the retrieved 3D waveforms. In addition, it is important to note how the mentioned error causes the integrated velocities to be significantly dependent on the number of observations used in the estimation process (i.e. the slope of the receiver movements varies according to observations number). This effect is already appreciable in the orange curve (1 meter error) and becomes clearly notable looking the blue curve (10 meters error) in figure 5.9.

**Effects on satellites coordinates**

It can be interesting to show that, although the uncertainties in the receiver position, satellites coordinates do not change. At first, this can look misleading. In fact, as it was expressed by the algorithm described at page 65, it appears clear that different receiver coordinates will lead to a different value for the receiver clock offset. This, in principle, should cause satellites coordinates to be computed for a different time epoch $t$ and, hence, to change. However, a quantitative
5.1. VADASE solutions for a simple simulation: geometric ranges

...analysis of involved differences will clarify this odd condition.

Let define $\delta t_{r0,i}$ as the reference value for the receiver clock offset obtained at epoch $i$ with the “correct” receiver position. Indicate $\delta t_{r1,i}$ and $\delta t_{r10,i}$ as the values obtained for epoch $i$ but using receiver position altered by 1 and 10 meters, respectively.

The difference of these values with respect to the reference was computed for every epoch. Table 5.7 shows the amount of the average of these differences.

<table>
<thead>
<tr>
<th>$n = 12000$</th>
<th>$\Delta$Clock [s]</th>
<th>$\Delta$Clock [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{n} \cdot \sum_{i=1}^{n} (\delta t_{r0,i} - \delta t_{r1,i})$</td>
<td>-1.95823993e-09</td>
<td>-0.59</td>
</tr>
<tr>
<td>$\frac{1}{n} \cdot \sum_{i=1}^{n} (\delta t_{r0,i} - \delta t_{r10,i})$</td>
<td>-1.95824116e-08</td>
<td>-5.87</td>
</tr>
</tbody>
</table>

Table 5.7: Differences on receiver clock offset due to receiver coordinates uncertainty

It is clear that the uncertainties in the receiver coordinates introduce a difference in the epochs at which satellites coordinates are computed. However, the amount of these differences is too small to reflect in a change in satellites position$^4$. Hence, satellites coordinates do not change (but at sub-millimeter level).

5.1.3 Effects of a possible error in the geometry model

Here, it is interesting to assess how an error in modeling the geometric configuration of the system (e.g. mistakes in satellites coordinates computation) will reflect in terms of waveforms retrieved with the variometric algorithm. With this aim, let maintain the “correct” position for the receiver and let process the simulated data not accounting for the receiver clock offset.

This corresponds to neglect step 2 of algorithm described at page 65, thereby resulting again in a different time epoch $t^s,i$ for satellites coordinates computation. Such a mistake in the model will cause significant differences in satellites positions. Table 5.8 reports the differences, averaged over 12000 epochs, for each satellite with respect to the correct model.

Related effects in terms of receiver waveforms achieved using VADASE are shown in figure 5.10.

On the left-hand y axis the red curve represents receiver displacements obtained using the correct model, whereas the blue curve results from not accounting

$^4$Recall that, generally speaking, satellites travel with a speed of about 3 km/sec
5.1. VADASE solutions for a simple simulation: geometric ranges

Table 5.8: Differences in satellites coordinates not accounting for receiver clock offset

<table>
<thead>
<tr>
<th></th>
<th>Δ X</th>
<th>Δ Y</th>
<th>Δ Z</th>
<th>Δ 3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>G01</td>
<td>2.082</td>
<td>0.767</td>
<td>−19.341</td>
<td>19.468</td>
</tr>
<tr>
<td>G03</td>
<td>6.616</td>
<td>3.693</td>
<td>17.434</td>
<td>19.009</td>
</tr>
<tr>
<td>G06</td>
<td>3.788</td>
<td>3.256</td>
<td>18.634</td>
<td>19.292</td>
</tr>
<tr>
<td>G14</td>
<td>−5.756</td>
<td>−0.925</td>
<td>−18.308</td>
<td>19.213</td>
</tr>
<tr>
<td>G16</td>
<td>−8.820</td>
<td>7.771</td>
<td>14.008</td>
<td>18.287</td>
</tr>
<tr>
<td>G19</td>
<td>12.721</td>
<td>7.627</td>
<td>9.334</td>
<td>17.525</td>
</tr>
<tr>
<td>G20</td>
<td>6.213</td>
<td>−2.763</td>
<td>−17.856</td>
<td>19.107</td>
</tr>
<tr>
<td>G23</td>
<td>15.388</td>
<td>6.742</td>
<td>−2.476</td>
<td>16.982</td>
</tr>
<tr>
<td>G31</td>
<td>0.513</td>
<td>13.575</td>
<td>−11.592</td>
<td>17.859</td>
</tr>
</tbody>
</table>

for the receiver clock offset. The former case leads to a 3D receiver displacement of 0.003 m in 10 minutes, which is explained by the receiver noise. The latter case returns a 3D displacement of 0.562 m, which is due to the errors in the geometry term introduced by wrong satellites coordinates values. On the right-hand y axis the number of observations used for the estimation is reported in green.

Comparing the outcomes of the described error in modeling the geometry term with the results coming from introducing an uncertainty in the receiver initial position, a first remark can be done. Here, in fact, the trend exhibited in the receiver displacements does not change according to the number of observations, as it was shown when introducing an uncertainty level of ∼ 17 meters on the receiver 3D initial position (see blue curve in figure 5.9).

Remarks

In this section, the geometry term of the variometric algorithm has been described and assessed. Moreover, the algorithm used for satellites coordinates was reported and, in this respect, the effects of possible modeling errors have been shown.

At first, receiver “correct” coordinates were altered by a certain quantity to investigate VADASE dependency on the errors in receiver position. In fact, this case would better reproduce real life situations in which receiver initial position always suffers from inaccuracy. Table 5.6 shows that the uncertainty introduced in the geometry term from the receiver side reflects on the achieved final 3D
5.2 Retrieve receiver movements

In the previous section, data simulated for a static receiver have been processed to assess VADASE effectiveness and consistency in modeling the geometric terms. Interestingly, it was shown that the inaccuracies produced some differences in receiver clock error computation. The amount of these differences, however, was too low to affect satellites coordinates, which, in fact, remained unchanged.

Then, in order to investigate the impact of possible errors in modeling the geometry term, satellites coordinates were computed not accounting for the receiver clock error. This produced mistakes of about 20 meters in the 3D satellites position (table 5.8) thereby introducing errors in the variometric algorithm that led to a receiver 3D displacement of 0.562 m.

5.2 Retrieve receiver movements

Figure 5.10: Receiver waveforms (integrated velocities) using GPS constellation for 10 minutes interval: effects of neglecting receiver clock offset. Green dots show the number of observations used in the least squares solution. Data are obtained from Spirent simulator using the scenario described in table 5.1.
of the variometric algorithm (i.e. $e_s^r$ and $[\Delta \rho_s^r]_{OR}$). Prior to continue considering other real effects in the variometric model, in order to reach its complete formulation (equation 4.7), the simple form described in equation 5.1 has been further tested in order to investigate its capacity to retrieve a certain receiver motion.

The example produced in this second scenario figures a receiver not being static but moving in a circle. The complete set of used parameters is listed in table 5.9.

<table>
<thead>
<tr>
<th>SCENARIO PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date — Rate</td>
</tr>
<tr>
<td>Position</td>
</tr>
<tr>
<td>Circular motion</td>
</tr>
<tr>
<td>Tropospheric delay</td>
</tr>
<tr>
<td>Ionospheric delay</td>
</tr>
</tbody>
</table>

Table 5.9: Scenario 2: circular receiver motion

Javad Delta receiver was used for data acquisition, both GPS and Galileo constellations were simulated. However, because of the problems concerning Galileo and already described in section 5.1, only GPS results will be reported from now onwards.

Figure 5.11 shows a detail of the retrieved receiver waveforms. Both East and North components display a sinusoidal behaviour with an amplitude of 4 meters and a period of $2\pi$ s. This corresponds exactly to the conditions applied by Spirent simulator. Since the motion was operated only on planimetric components, the vertical one just shows the influence of the receiver noise on the retrieved waveforms.

Figure 5.12 presents a 2 dimensional view of the circular receiver motion.
5.2. Retrieve receiver movements

Figure 5.11: Receiver waveforms (integrated velocities) using GPS constellation: circular receiver motion for a 30 seconds interval. Data are obtained from Spirent simulator using the scenario described in table 5.9
5.2. Retrieve receiver movements

Figure 5.12: Receiver circular motion
5.3 Satellites clock errors

Scenarios 1 and 2, described in sections 5.1 and 5.2, assessed the effectiveness in modeling the geometric terms of the simple variometric model (equation 5.1) under two different conditions. The former consisted on a receiver remaining static in its initial position while the latter simulated a receiver undergoing a circular motion.

For the first case, VADASE algorithm demonstrated its validity retrieving a 3D receiver displacement of 3 millimeters (which corresponds to the level of the receiver and observations noise) after 10 minutes (20 Hz acquisition rate). In addition, a sensitivity analysis showed the dependency of the results on the correct knowledge of initial receiver position. Further, an example of a wrong model for satellites coordinates computation has been shown.

For the second case, VADASE proved its capacity to reconstruct the characteristics of the motion simulated for the receiver.

In scenario number 3, described in this section, Spirent simulator was fed with a RINEX navigation file (brdc2830.11n) containing real GPS broadcast parameters including satellites clock error (which were not considered up to now). The variometric algorithm was expanded to include the term accounting for the satellite clock error ($\delta t^s$), as follows in equation 5.3.

$$[\lambda_1 \Delta \Phi^s_r]_{L1,L2} = (e_r^s \cdot \Delta \xi_r + c \Delta t^s_r) + \left( |\Delta \rho^s_{OR} - c \Delta t^s_r| + \Delta \epsilon^s_r \right)$$  (5.3)

Regarding the stochastic model, since the atmosphere was not considered in the scenario, all observations were evenly weighted. The least square estimation of the 3D velocity was based upon the entire set of the variometric equation 5.3 which can be written (separately for L1 and L2) for each satellite in common to two generic consecutive epochs $(t, t + 1)$.

Septentrio PolarRx3 receiver was used to acquire the observations with 20 Hz rate. The receiver remained fixed in its initial position; all the parameters describing scenario 3 are summarized in table 5.10.

5.3.1 Modeling satellite clock error

The well known equation 5.4, described in [44], allows to compute the satellite clock error with respect to the GPS coordinate time at epoch $t$, using the parameters included in the broadcast ephemerides.

$$\delta t^s(t) = a_0 + a_1 (t - t_{oe}) + a_2 (t - t_{oe})^2 + \Delta t_r$$  (5.4)
5.3. Satellites clock errors

<table>
<thead>
<tr>
<th>SCENARIO PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date — Rate</td>
</tr>
<tr>
<td>Position</td>
</tr>
<tr>
<td>Tropospheric delay</td>
</tr>
<tr>
<td>Ionospheric delay</td>
</tr>
</tbody>
</table>

Table 5.10: Scenario 3: using real broadcast ephemerides

where $a_0$ [s] is the satellite clock bias, $a_1$ [s/s] is the satellite clock drift, $a_2$ [s/s$^2$] is the satellite frequency drift, $t_{cr}$[s] is the clock data reference time, $\Delta t_r$[s] is the correction due to relativity effects and $t$ is the current time epoch.

Since the broadcast clock parameters are predicted, some residual errors remain. These residual clock errors can result in ranging errors that typically vary from 0.3$\sim$4 meters, according to the type of satellite and the age of the broadcast data [45, p. 304].

To account for the satellite clock error at epoch $t$, the model described in equation 5.4 is used in the variometric algorithm and, hence, it was implemented in VADASE software. Receiver waveforms obtained processing data simulated for the present scenario are reported in figure 5.13.

On the left-hand y axis the red curves present the receiver displacement for East North and Up components. The green curves, scaled on the right-hand y axis, show the number of observations used by the variometric algorithm to solve the least squares system. After 15 minutes (20 Hz acquisition rate) the 3D receiver displacement reaches 0.005 m. This value is explained considering the receiver noise.

A deeper analysis was carried on to assess the impact of $\Delta t_r$. This term is used to correct the satellite clock error for the relativistic effect that arises because of the orbits eccentricity. When the satellite is at perigee, the satellite velocity is higher and the gravitational potential is lower — both cause the satellite clock to run slower. When the satellite is at apogee, the satellite velocity is lower and the gravitational potential is higher — both cause the satellite clock to run faster [45, p. 306]. To compensate for this effect, the following equation is used

$$\Delta t_r = F e \sqrt{A} \sin E_k$$

(5.5)

where $F = -4.442807633 \times 10^{-10}$[m$^3$/s$^2$], $e$ is the satellite orbital eccentricity, $A$ is the semimajor axis of satellite orbit and $E_k$ is the eccentric anomaly of the satellite orbit.
5.4. Modeling the atmospheric effects

Figure 5.13: Receiver waveforms (integrated velocities) using GPS constellation for a 15 minutes interval: modeling satellite clock error. Data are obtained from Spirent simulator using the scenario described in table 5.10

To give an example of the amount of this relativistic term, satellites clock errors were computed not accounting for $\Delta t_r$. Table 5.11 reports the values of the differences stemming from computing satellites clock error with or without taking into consideration the described relativistic effect. For each satellite, the differences were averaged over 18000 epochs (i.e. observations number within the considered time interval).

As already described in the former section, the presence of a bias in the variometric model reflects as a drift in the receiver displacements. The wrong compensation for satellites clocks, obtained without considering the relativistic correction term, resulted in a 3D receiver displacement of 1.826 meters and introduced significant drifts in the receiver waveforms (blue curve in figure 5.14).

5.4 Modeling the atmospheric effects

All the scenarios described in the former sections were designed specifically to test the geometry terms (sections 5.1, 5.2) and the satellite clock error term (sec-
5.4. Modeling the atmospheric effects

<table>
<thead>
<tr>
<th>18000 epochs</th>
<th>ΔClock [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sat Id</td>
<td></td>
</tr>
<tr>
<td>G02</td>
<td>3.712</td>
</tr>
<tr>
<td>G04</td>
<td>−3.003</td>
</tr>
<tr>
<td>G05</td>
<td>−1.702</td>
</tr>
<tr>
<td>G07</td>
<td>3.099</td>
</tr>
<tr>
<td>G08</td>
<td>4.713</td>
</tr>
<tr>
<td>G10</td>
<td>−7.064</td>
</tr>
<tr>
<td>G17</td>
<td>−3.301</td>
</tr>
<tr>
<td>G26</td>
<td>7.949</td>
</tr>
<tr>
<td>G27</td>
<td>−5.376</td>
</tr>
<tr>
<td>G28</td>
<td>−11.785</td>
</tr>
</tbody>
</table>

Table 5.11: Differences in satellites clocks not compensating for the relativistic term

This section intends to extend the variometric algorithm towards the consideration of signal propagation biases caused by the atmospheric layers that surround the Earth (i.e. troposphere and ionosphere). It is possible to differentiate the atmosphere around the Earth into different layers according to their physical properties and influences on the electromagnetic waves. With respect to the electromagnetic structure, the atmosphere is divided into the troposphere (or neutral atmosphere) and the ionosphere [42, p. 65]. In this section, a brief description of these two layers is given. Moreover, the impacts of the related atmospheric delays on the variometric algorithm are assessed.

Before entering the details of atmospheric biases, it might be useful to briefly recall some fundamentals concerning electromagnetic waves propagation through a generic medium. Waves propagation in a medium depends on the refractive index $n$ which is defined as the ratio between the speed of light in vacuum ($c$) and the propagation velocity of the signal through the medium ($v$)

$$ n = \frac{c}{v} \tag{5.6} $$

Equation 5.6 yields a refractive index equal to unity for vacuum. At the same
5.4. Modeling the atmospheric effects

Figure 5.14: Receiver waveforms (integrated velocities) using GPS constellation for a 15 minutes interval: impact of the relativistic satellite clock term. Data are obtained from Spirent simulator using the scenario described in table 5.10

time, a refractive index greater than 1 implies that electromagnetic waves are delayed compared to the time needed to travel the same distance in vacuum with the speed of light. Refractive indexes depend on water vapor, temperature, pressure and amount of free electrons. Further, a medium is called dispersive if the propagation speed (or the refractive index) is a function of the wave’s frequency. In a dispersive medium, the propagation velocity of the signal carrier phase differs from the velocity associated with the waves carrying the signal information. These are the so called phase velocity \( v_{ph} \) and group velocity \( v_{gr} \), respectively.

5.4.1 Tropospheric effect

The perturbation caused to signal propagation by the neutral (i.e. the not ionized part) atmosphere is referred to as tropospheric refraction or tropospheric delay. Although this name might not be complete (i.e. it neglects the contribution of the stratosphere, which is another constituent of the neutral atmosphere), it can be justified since the troposphere gives the dominant contribution to the overall
5.4. Modeling the atmospheric effects

delay.

The troposphere extends from the Earth’s surface up to an altitude of approximately 50 km [42, p.65]. It is a non dispersive medium with respect to waves up to frequencies of 15 GHz [42, p. 128]. Hence, GNSS signals propagation through the neutral atmosphere is frequency independent. This yields an immediate drawback: it is not possible to eliminate the tropospheric delay combining observations from two different frequencies.

This delay varies according to the tropospheric refractive index which is dependent on the local temperature, pressure and relative humidity. The overall tropospheric delay can be further distinguished in two separate components:

- a dry part, which accounts for $\sim 90\%$ of the total delay and is accurately predictable
- a wet part, which accounts for the remaining delay (i.e. $\sim 10\%$) that is caused by the water vapor. This part is very difficult to predict because water vapor distribution is highly mutable with respect to both time and space

If left uncompensated, the overall delay would cause an equivalent range delay varying between $2.3 \div 2.5$ meters, according to satellite elevation angle [42].

A new simulation was devoted to investigate the impact of the tropospheric delay on the variometric algorithm. The parameters used for the new scenario (i.e. scenario 4) are resumed in table 5.12.

<table>
<thead>
<tr>
<th>SCENARIO PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date — Rate</td>
</tr>
<tr>
<td>Position</td>
</tr>
<tr>
<td>Tropospheric delay</td>
</tr>
<tr>
<td>Ionospheric delay</td>
</tr>
</tbody>
</table>

Table 5.12: Scenario 4: modeling the tropospheric delays

It might be immediately noted that the acquisition rate is 50 Hz (Javad Delta receiver was used) and that the tropospheric delay is simulated by Spirent using the STANAG model. This model, described in details in [59], defines the range error due to the troposphere as follows

$$R(h, Z_r) = f(Z_r) \cdot \Delta R(h)$$ (5.7)

where
5.4. Modeling the atmospheric effects

\[ R(h, Z^s_r) = \text{Total range error} \quad [m] \]
\[ \Delta R(h) = \text{Range error as a function of altitude} \quad [m] \]
\[ h = \text{Altitude above mean sea level} \quad [m] \]
\[ Z^s_r = \text{Elevation angle between satellite and receiver} \]
\[ f(Z^s_r) = \text{Range error factor (mapping function)} \]

with

\[ f(Z^s_r) = \frac{1}{\sin(Z^s_r) + 0.00143 \left( \tan(Z^s_r) + 0.0455 \right)} \quad Z^s_r < 90^\circ \]
\[ f(Z^s_r) = 1 \quad Z^s_r = 90^\circ \]

The total range error is computed as the sum of the contributes of three layers, as follows

\[ \Delta R(h) = \Delta R_1(h) + \Delta R_2(h) + \Delta R_3(h) \quad [m] \]

where

\[ \Delta R_1(h) = \text{Range error for altitude} \quad 0 \ km \leq h \leq 1 \ km \]
\[ \Delta R_2(h) = \text{Range error for altitude} \quad 1 \ km < h \leq 9 \ km \]
\[ \Delta R_3(h) = \text{Range error for altitude} \quad 9 \ km < h \leq h_{sv} \]

and \( h_{sv} \) is the altitude of the satellite. The surface refractivity index at mean sea level is 324.8. For the complete formulation of each contribute of range error refer to [59].

The variometric algorithm, extended to include the tropospheric term, results as follows

\[ [\lambda \Delta \Phi^s_r]_{L1,L2} = (e^s_r \cdot \Delta \xi_r + c \Delta \delta t_r) + [\Delta \rho^s_{OR}] + \Delta T^s_r + \Delta \epsilon^s_r \quad (5.8) \]

Here, it should be underlined that, how it was shown in equation 4.5, the variometric model implemented in VADASE software accounts for the tropospheric delay \( \Delta T^s_r \) using the Saastamoinen troposphere model.

Regarding the stochastic model, the observations were weighted by the squared cosine of the zenith angle between the satellite and the receiver \( Z^s_r \).

\[ w = \cos^2(Z^s_r) \]

The least squares estimation of the 3D velocity was based upon the entire set of the variometric equation 5.8 which can be written (separately for L1 and L2) for each satellite in common to two generic consecutive epochs \((t, t + 1)\). The receiver waveforms are reported in figure 5.15.
After 10 minutes (which here correspond to 30000 epochs) the 3D receiver displacement reaches 0.027 m. Moreover, the North and the Up components of the waveforms display a drift which is clearly dependent on the number of observations used in the estimation process (which are plotted as green dots scaled on the right-hand y axis).

To prove that the noted behaviors and the amount of the final displacements are due to the differences between the models used to simulate and to solve the tropospheric delay (i.e. STANAG and Saastamoinen models, respectively), the STANAG model was implemented in VADASE software. The implementation was tested on the same scenario and the results are displayed in figure 5.16.

The blue curves show the receiver waveforms obtained with the implemented STANAG tropospheric model whereas the red curves refer to the results of the Saastamoinen model. In this case (blue curves), the receiver 3D displacement achieved after 10 minutes is 0.003 m. Further, the waveforms do not show any dependency on the observations number.
5.4. Modeling the atmospheric effects

Figure 5.16: Receiver waveforms (integrated velocities) using GPS constellation for a 10 minutes interval: comparing Saastamoinen and STANAG tropospheric models. Data are obtained from Spirent simulator using the scenario described in table 5.12

Remarks

In this section, the results of a new simulation, devoted to investigate the effect of the tropospheric delay on the variometric algorithm, have been described. Spirent simulator was instructed to apply the STANAG model to map the tropospheric delays into related ranges errors for each satellite.

The variometric model was extended (equation 5.8) to include the tropospheric term $\Delta T_r$, which was formerly computed using Saastamoinen troposphere model. Because of the differences between the model used to simulate the tropospheric delay (i.e. STANAG) and the model used in the variometric algorithm (i.e. Saastamoinen), some residual errors for the troposphere remain in the estimated 3D velocities. This leads to a 3D receiver displacement of $\sim 0.030$ m after 10 minutes (50 Hz acquisition rate). Moreover, all the three components of the displacements showed to be significantly dependent on the number of observations used to solve the least squares problem, as it is shown in figure 5.15.
When VADASE uses the same model applied by Spirent (i.e. STANAG), the receiver 3D displacement achieved after 10 minutes is 0.003 m and no drift is present in any of the three waveforms (i.e. East, North and Up). The results of this latter example are shown in figure 5.16 using blue curves.

5.4.2 Ionospheric effect

The ionosphere is the region of the atmosphere located approximately between 70 km and 1000 km above the Earth’s surface. Within this region, ultraviolet rays from the sun ionize a portion of the gas molecules and release free electrons. The influence of the ionosphere layer region over electromagnetic waves propagation varies according to the density of these free electrons, which is expressed in terms of TEC. The TEC is given in TEC Units (TECU), with

\[ 1 \text{ TECU} = 10^{16} \text{ electrons per m}^2 \]

and depends on the solar activity, diurnal and seasonal variations, and the Earth’s magnetic field.

Ionosphere is a dispersive medium with respect to GNSS signals. In details, carrier waves propagate with phase velocity whereas for code measurements group velocity has to be considered. Starting from equation 5.6 and using Rayleigh equation for phase and group velocity (details in [42, pp. 116, 117]) it is possible to derive the refractive indexes for carrier phase and pseudorange propagating in the ionosphere

\[ n_{ph} = 1 + \frac{c_2}{f^2} \quad n_{gr} = 1 - \frac{c_2}{f^2} \]  

(5.9)

where \( c_2 \) is estimated as \( c_2 = -40.3N_e^5[H z^2] \). As a consequence of equation 5.9, during the propagation through the ionosphere, a group delay and a phase advance occur for GNSS signals. This phenomenon is referred to as ionospheric divergence and causes the measured carrier phase to be shorter and the code pseudorange to be longer compared to the geometric range from the satellite to the receiver. Here, it is important to underline that the amount of the difference is the same in both cases, only the sign is different.

If left uncompensated, the ionospheric delay can lead to an equivalent range delay up to 100 meters [46]. The most effective way to remove the ionospheric delay is to combine two signals with different frequencies. This dual-frequency combination is the main reason why GNSS satellites emit (at least) two carrier

---

\(^5N_e = \text{Electron density}\)
5.5. Addressing rapid and sharp changes in atmospheric conditions

The effect of the ionospheric delay was reproduced in a new scenario whose parameters are listed in table 5.13. The receiver remained fixed in the initial position; data were recorded using Javad Delta receiver at 20 Hz frequency.

<table>
<thead>
<tr>
<th>SCENARIO PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date — Rate</td>
</tr>
<tr>
<td>Position</td>
</tr>
<tr>
<td>Tropospheric delay</td>
</tr>
<tr>
<td>Ionospheric delay</td>
</tr>
</tbody>
</table>

Table 5.13: Scenario 5: ionospheric delay

Then, to address the ionospheric delay, the ionosphere-free combination was implemented in the variometric algorithm, as follows

\[
\alpha [\lambda \Delta \Phi^s_r]_{L1} + \beta [\lambda \Delta \Phi^s_r]_{L2} = (e^s_r \ast \Delta \xi_r + c \Delta \delta t_r) + [\Delta \rho^s_r]_{OR} + \Delta \epsilon^s_r \tag{5.10}
\]

where \(\alpha = f^2_{L1} / (f^2_{L1} - f^2_{L2})\) and \(\beta = -f^2_{L2} / (f^2_{L1} - f^2_{L2})\) are the standard coefficients of the ionosphere-free combination. The results in terms of receiver waveforms are reported in figure 5.17.

On the left-hand y axis the red curves represent the receiver displacements in the three components; the green curves, scaled on the right-hand y axis, show the observations used to solve the least squares estimation problem. The ionosphere-free combination implemented to eliminate the ionospheric delay yields a final 3D displacement of 0.005 m after 15 minutes. If not compensated, the ionospheric delay would have led a 3D displacement of 0.404 m for the same time interval.

5.5 Addressing rapid and sharp changes in atmospheric conditions

Sudden changes in atmospheric conditions, such as ionosphere scintillation, Traveling Ionospheric Disturbances (TID) (also referred to as ionospheric fronts), and rapid variations of water vapor content in the troposphere (also called tropospheric fronts), represent a major threat to real-time GNSS positioning algorithms and to availability and continuity of navigation and communication services.

One of the major difficulty is that, so far, these events are, spatially and temporally unpredictable. Moreover, the physical behavior of a such complex...
5.5. Addressing rapid and sharp changes in atmospheric conditions

mean as the atmosphere has to be understood and studied in more details in order to duly comprehend the mechanisms that provoke such effects. Ever since these issues will be solved, it will be possible to assess tuned models in order to effectively account for these rapid changes of atmospheric conditions.

On the other hand, these perturbations are not evenly distributed in each of the points where the signals from the visible satellites cross the atmosphere. Hence, whenever one of the mentioned disorder occurs, it is likely to affect one or at most a few satellites at a time, thus leaving the possibility to exploit the plain signals.

The Spirent GNSS simulator does not directly feature the capability to simulate any of the presented effects. However, it allows the user to modify the amplitude and the phase of the signals that are being simulated. This is made possible using duly formatted text files (referred to as User command file) that can be uploaded in the framework of the scenario selected for the simulation.

In the sense that the scenario parameters do not take into consideration neither the ionospheric scintillation nor TID nor rapid changes in water vapor content.

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Figure 5.17: Receiver waveforms (integrated velocities) using GPS constellation for a 10 minutes interval: addressing ionospheric delay. Data are obtained from Spirent simulator using the scenario described in table 5.13
Exploiting this feature, the Navigation department of the Navigation and Communication Institute, DLR, Munich, has implemented a bunch of Matlab functions that generate *User command files* in order to instruct Spirent to alter the simulated signals as if one of the mentioned disturbances was going on.

Taking advantage from this possibility, three scenarios were designed to simulate different severe atmospheric events, namely:

1. ionospheric scintillation
2. rapidly propagating ionospheric front
3. rapidly propagating tropospheric front

The generated observations were collected in RINEX files and processed using the variometric algorithm. The results obtained are discussed in the next sections together with the strategies adopted, and implemented, in order to address these effects in real-time.

### 5.5.1 Ionospheric scintillation

Irregularities in the ionospheric layer of the Earth’s atmosphere can, at times, lead to rapid fading in received signal power levels [45, p. 295]. This phenomenon, usually referred to as ionospheric scintillation, causes a perturbation to both the receiver signal amplitude and phase and can lead to a receiver being unable to track one or more satellites for a short period of time.

Scintillation occurs most frequently during the peak of the solar cycle. In addition, it may be more severe in equatorial regions after the sunset and, to a lower extent, in polar and auroral regions. Typically, scintillation has minimum impact in mid-latitude regions [39].

From a mathematical standpoint, the fluctuations of the signal power ($\delta P$) caused by scintillation can be modeled following a so called Nakagami-m probability density function (whose complete form can be found in [45, 39]), whereas for phase fluctuations it is used a zero-mean Gaussian distribution. The strength of this power fluctuation is expressed by means of the so called $S_4$ index, which is equal to the standard deviation of $\delta P$. The $S_4$ index can not exceed $\sqrt{2}$.

Using a bundle of Matlab functions that implement the Nakagami-m distribution, the amplitude fading and phase changes corresponding to a moderate scintillation (i.e. $S_4 = 0.5$) were generated for a 20 minutes time period and stored in a *User command file*. Then, this file was uploaded in Spirent simulator.
5.5. Addressing rapid and sharp changes in atmospheric conditions

in order to alter signals amplitude and phase broadcast from one GPS satellite (i.e. \textit{G03}). The receiver remained fixed in its initial position, all general parameters adopted in the scenario are summarized in table 5.14.

<table>
<thead>
<tr>
<th>SCENARIO PARAMETERS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Date — Rate</td>
<td>Oct-10-2011 00:20:00 — 15 min — 20 Hz</td>
</tr>
<tr>
<td>Position</td>
<td>$X = 6378137.0$, $Y = 0.0$, $Z = 0.0$</td>
</tr>
<tr>
<td>Tropospheric delay</td>
<td>Absent</td>
</tr>
<tr>
<td>Ionospheric delay</td>
<td>Scintillation ($S_4 = 0.5$)</td>
</tr>
</tbody>
</table>

Table 5.14: Scenario 6: ionospheric scintillation

In this case, the least squares estimation of the 3D velocity was based upon the entire set of the variometric ionosphere-free equation 5.10 that can be written for each satellite in common to two generic consecutive epochs ($t$, $t + 1$).

RINEX format definitions allow to map the signal to noise ratio into values from 0 to 9. A value of 0 means either bad observations or unknown signal to noise ratio, a value of 9 corresponds to the maximum signal to noise ratio. Figure 5.18 shows the raw signal strength as reported in the RINEX observations file for both frequencies of GPS satellite \textit{G03}.

As it is shown in figure 5.18, raw signal strength for L2 carrier phase degrades significantly when the scintillation starts to take action on satellite \textit{G03}. Moreover, for several epochs the receiver is not able to track the L2 carrier frequency at all. On the other hand, L1 signal strength does not change considerably. This is because the $S_4$ index is a function of carrier frequency ($S_4 \sim 1/f^{1.5}$), therefore, when fading due to ionospheric scintillation occurs, the observed $S_4$ index for a signal on L2 is approximately 1.45 times greater than $S_4$ on L1 [45]. As a result of this dependency, scintillation would more likely cause outages for GPS signals on L2 than for L1 signals.

The generated data were processed with VADASE software and the estimated 3D velocities are displayed in figure 5.19.

The red curves show the receiver velocity in each component (East, North, Up) whereas the green curves show the number of observations used to solve the least squares estimation problem. A primary effect of the scintillation can be seen in the variability of the observations number, due to loss of lock of the L2 carrier phase observations of satellite \textit{G03}.

As it was expected, the receiver showed no displacements ever since the scintillation effect started. Then, the variometric algorithm reacted immediately: the level of noise in the solutions increased by one order of magnitude in each component, thereby severely compromising the possibility to detect any receiver
5.5. Addressing rapid and sharp changes in atmospheric conditions

Figure 5.18: Raw signal strength for satellite G03: effects of the signal power fading (mainly on L2 frequency) caused by ionospheric scintillation

So far, it has been shown that a scintillation in the ionosphere, even though interesting only observations of 1 satellite at a time, can compromise the effectiveness and the accuracy of the variometric algorithm in detecting the receiver displacements between two consecutive epochs. Now, in order to strengthen VA-DASE with respect to the described perturbation, and more generally to all the perturbations that threaten the real-time capabilities of the algorithm, efforts were made in the direction of recognizing and removing the spoiled observations from the epoch by epoch displacement solution.

To this aim, a data snooping technique was designed, tested and implemented (even though not yet in a complete way at the moment of this writing) on the basis of the theoretical background described in [8, 77]. Here, the description of this technique, given in the following, limits to the statistical test used to detect the observations to be excluded from the single variometric epoch. Afterwards, it will be given an important remark about the general workflow implemented in the software in order not to compromise its real-time capabilities.
5.5. Addressing rapid and sharp changes in atmospheric conditions

Figure 5.19: Receiver estimated velocities using GPS constellation for a 15 minutes interval: effects of the ionospheric scintillation. Data are obtained from Spirent simulator using the scenario described in table 5.14

Data snooping technique: statistical test

Considering dual frequency carrier phase observations from the $n$ satellites in view at epochs $t$ and $t+1$ let form the variometric ionosphere-free observation $VAR_1$. Then, let compute the difference between the variometric observation and the so called “Known term” (the procedure was already described in section 4.3.1). This quantity (later referred to as the array $L_K$) will be tested to detect possible observations to be excluded from the least squares solution. The test can be described by means of the following pseudocode:

1. compute the mean value ($\mu$) of $L_K$
2. for each element $l_{ki}$ in $L_K$, evaluate $|l_{ki} - \mu|$ and compute the maximum ($max_i$)
3. if $max_i$ is bigger than threshold, go to 4; else go to 6
4. exclude observation $i$ from the array $L_K$, generating a new array referred to as $L_{K1}$; compute the mean value ($\mu_1$) of $L_{K1}$ and the standard deviation value ($\sigma_1$) of $L_{K1}$
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5. if $\max_i$ is bigger than $\mu_1 + a \cdot \sigma_1$, exclude observation $i$ from the variometric epoch and start over with step 1; else, go to 6

6. proceed to least squares estimation of the receiver velocity

where $a$ is the factor used to define the interval boundary that determines the observation exclusion. $\text{threshold}$ is a value used to trigger the algorithm thereby avoiding that all the variometric epochs undergo the test and that plain observations are removed. In details, this value is tuned according to the acquisition data rate and depends on the level of noise in the observations. Generally speaking, the higher the interval between two consecutive observations the higher the effect of any disturbance. Therefore, the higher is the acquisition rate the lower will be the value chosen for $\text{threshold}$.

The described DS technique was implemented in VADASE software and tested in the case of the ionospheric scintillation. The results achieved by the enhanced variometric algorithm are exposed in figure 5.20.

Figure 5.20: Receiver estimated velocities using GPS constellation for a 15 minutes interval: addressing ionospheric delay through a data snooping technique. Data are obtained from Spirent simulator using the scenario described in table 5.14
The estimated receiver velocities appear not to be influenced by the effect of the ionospheric scintillation. In fact, the standard deviation of the solutions is in the range of $0.0003 \div 0.0005$ m for the horizontal components and reaches 0.001 m for the vertical one.

Implementation remark

As it was already mentioned, the algorithm is not yet duly implemented. In fact, it might happen that the test removes one observation from the variometric epoch formed between $t - 1$ and $t$ but it leaves the same observation in the variometric epoch formed between $t$ and $t + 1$. The effect is clearly shown in figure 5.20 where the satellite $G03$, even though suffering from the scintillation effect, is not removed from all the consecutive variometric epochs (i.e. within the interval when the scintillation acts, the observation number, showed in green, still displays some jumps). This issue compromises the complete effectiveness of the data snooping technique and causes some major drawbacks when integrating the velocities to obtain the receiver movements. As a matter of fact, this displays only as a technical implementation problem. Several strategies to solve it have already been discussed and are being currently introduced in VADASE software. More importantly, the described data snooping technique proved to strengthen the variometric algorithm resulting in velocities estimations that are not altered by an average (i.e. $S_4 = 0.5$) ionospheric scintillation event.

5.5.2 Ionospheric front

Ionosphere-free combination proved to be correctly implemented and to remove ionospheric biases up to the second order from VADASE solutions (5.4.2). However, communication and navigation systems relying on ray propagation through the atmosphere should be able to compensate for the effects of sharp changes (e.g. 90% reduction) in TEC associated with the ionospheric trough and storm-time disturbance effects at mid-latitude and high latitude. TID can also affect precise GPS positioning. TID commonly occur in winter and autumn, mostly during daytime. Their wavelengths are about 50-500 km and they travel at speeds of 5 to 10 km per minute [1]. As it was already shown for the ionospheric scintillation, sudden changes in ionosphere conditions represent a major threat for real-time GNSS algorithms.

A new scenario was designed with the aim to test VADASE under the effect of a front of ionosphere perturbation moving with defined characteristics and disturbing satellites according to their positions with respect to the receiver. The simulation was realized loading a User command file into Spirent simulator. This file contained a description of the epoch and the related ionospheric delay to be
applied to the observations generated for each satellite. Scenario’s parameters are described in table 5.15, an extract of the user command file is shown hereafter:

<table>
<thead>
<tr>
<th>Time</th>
<th>SVID</th>
<th>Type</th>
<th>Delay</th>
<th>Rate</th>
<th>Acc</th>
</tr>
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<td>88482</td>
<td>1</td>
<td>1</td>
<td>1.33425638e−008</td>
<td>0.000000000e+000</td>
<td>0.000000000e+000</td>
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</tr>
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<td>0.000000000e+000</td>
<td>0.000000000e+000</td>
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<td>0.000000000e+000</td>
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<td>0.000000000e+000</td>
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<td>-2.69831371e−010</td>
<td>1.12114631e−10</td>
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<td>0.000000000e+000</td>
</tr>
</tbody>
</table>

SCENARIO PARAMETERS

| Date — Rate | Oct-10-2011 00:20:00 — 20 min — 20 Hz |
| Position    | X = 6378137.0 Y = 0.0 Z = 0.0 |
| Tropospheric delay | Absent |
| Ionospheric delay | Ionospheric front effects |

Table 5.15: Scenario 7: ionospheric front

At epoch 88462 [second of the week] the ionospheric front already affected satellites G01, G03, G06, G14, G19 and G22, introducing an additional delay/advance\(^7\) of 4 meters in the observations. For satellite G20 the ramping effect started at epoch 88462 with a fixed rate and an acceleration. Within 24 seconds the ramping effect will be completed and the delay/advance will reach the constant level of 4 meters\(^8\).

To appreciate how the disturbance affects the observations it is possible to combine pseudoranges and carrier phases (at epoch t) in such a way that the ionospheric effect is highlighted. The so called Code Minus Carrier (CMC) observation (eq. 5.11), in fact, takes advantage of the ionospheric divergence, described at page 83. Using this combination, twice the effect of the ionospheric

---

\(^7\)The ionospheric effect will cause a delay in the code pseudoranges and and advance, of the same amount, in the phase carrier observations

\(^8\)4 meters is the result in length unit of a time interval of 1.33425638 \(\cdot 10^{-8}\) seconds
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delay ([m]) is displayed.

\[
CMC_i = P_{r,i}^s - L_{r,i}^s = \left( P_{r,i}^s - c (d\tau_r - d\tau^s) - T_{r,i}^s + I_{r,i}^s \right) + \\
- \left( P_{r,i}^s - c (d\tau_r - d\tau^s) - T_{r,i}^s - I_{r,s}^s - \lambda_i N_{r,i}^s \right) \\
= 2 I_{i,r}^s + \lambda_i N_{i,r}^s
\]  

(5.11)

where \( i \) refers to the chosen frequency (e.g. \( \lambda_1 \) or \( \lambda_2 \)), \( L_{r,i}^s \) is the carrier phase observation \( (L_{r,i}^s = \lambda_i \Phi_r^s) \) and all the other terms were already introduced in equation 4.1.

The skyplot observed by the receiver is displayed in figure 5.21.

---

**Figure 5.21: Scenario 7: skyplot**
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Figures 5.22 and 5.23 show the CMC combinations for both carrier phases L1 and L2.

![CMC for L1 carrier phase - Impact of iono ramp](image)

Figure 5.22: CMC: effect of the ionospheric ramp over L1

By looking at these figures, some preliminary annotations should be made:

- the effect of the ionospheric ramp is displayed only by satellites G01, G06, G14 and G20. This behaviour appears not to be coherent with the user input file supplied to Spirent. This, in fact, was supposed to simulate the ramping effect also for satellites G03, G19 and G22.

- L1 CMC combination (figure 5.22) shows very clearly how satellites observations are struck by the ionospheric front. When the front comes across a satellite, CMC combination grows rapidly until it reaches (in approximately 25 seconds) its final value, which corresponds to twice the imposed ionospheric delay (i.e. 8 meters).

- L2 CMC combination (figure 5.23) displays an unexpected behaviour. The evident jumps affecting satellites testify that data are not continuous. In addition, the front takes almost one minute to reach its maximum value of delay, which appears to be different for each satellite. Since the delay caused by the front should not be dependent on the frequency (i.e. the user
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Figure 5.23: CMC: effect of the ionospheric ramp over L2

input file has no information regarding the frequency), these significant differences with respect to L1 behaviour were not expected.

A deeper analysis performed looking into RINEX observations data shows that pseudorange and L2 carrier phase values become similar to each other after a data loss, as if the ionospheric ramp was not present (e.g. values for G20):

<table>
<thead>
<tr>
<th>sec of week</th>
<th>hh:mm:ss</th>
<th>C2 [m]</th>
<th>L2 [m]</th>
<th>CMC2 [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>88496.75</td>
<td>0:34:56.75</td>
<td>23262357.630</td>
<td>23262355.696</td>
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<td>88498.35</td>
<td>0:34:58.35</td>
<td>23261478.098</td>
<td>23261477.949</td>
<td></td>
</tr>
<tr>
<td>88506.30</td>
<td>0:35:06.30</td>
<td>23257107.284</td>
<td>23257104.434</td>
<td></td>
</tr>
<tr>
<td>88507.90</td>
<td>0:35:07.90</td>
<td>23256228.333</td>
<td>23256228.284</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.24 gives a visual representation of the problem for satellite G20. Here, it seems that data loss is due to Spirent simulator (and not to acquisition
problems in the receiver) that was not capable, in this occasion, to duly reproduce
the effect of a ionospheric front over both frequencies. As a matter of fact, the
outcome of the simulation was that L1 and L2 carrier phases displayed delays
that were not consistent with each other. Indeed, this looks quite different from
the expectations and, at the moment of this writing, new simulations are already
foreseen in order to run again the same scenario and fix the mentioned issues.
Nonetheless, generally speaking, the aim of the simulation was to assess VADASE
algorithm in case some satellites were exposed to sudden changes in ionosphere
conditions. Therefore, even though not duly representing a “pure” ionospheric
front event, the data were compliant with the prefixed goals and were thereby
processed.

In details, ionosphere-free combination was adopted to eliminate (or at least
reduce) disturbances in the estimated solutions. The least squares estimation of
the 3D velocity was based upon the entire set of equations 5.10 that could be
written for each satellite in common to two generic consecutive epochs \((t, t + 1)\).
The estimated receiver velocities are shown, with respect to the right-hand y axis,
in figure 5.25. At the same time, on the left-hand y axis the CMC observations
(L1 carrier frequency) of the satellites that came across the ionospheric front are
reported.

Figure 5.24: Problems in Ionospheric ramp simulation
5.5. Addressing rapid and sharp changes in atmospheric conditions

Figure 5.25: Receiver estimated velocities using GPS constellation for a 15 minute interval: traveling ionospheric disturbances. VADASE solutions are displayed on right-hand y axis. Code Minus Carrier observation for L1 carrier phase is displayed on the left-hand y axis. Data are obtained from Spirent simulator using the scenario described in table 5.15

At a glance, it is very clear that the algorithm behaves unreliably within the interval when the ionosphere ramping is in act. The receiver velocities jump in unpredictable ways around the expected value (i.e. no receiver displacement) and reach an offset up to several centimeters. As an additional remark it should be noted that, in real-field cases, these jumps could be very difficult to address and they might be easily misinterpreted as if the receiver was undergoing a real displacement.

The possibility to recognize and remove in real-time the observations altered by the ionospheric perturbations was tested using the variometric algorithm enhanced with the data snooping technique already described in section 5.5.1. The results are shown in figure 5.26.

The jumps in the estimated velocities are not present thanks to the effect of the data snooping technique. This was able to effectively identify and remove the observations that were “ruined” to a larger extent by the ionospheric disturbances. Overall, the present implementation proved to be robust with respect to
5.5. Addressing rapid and sharp changes in atmospheric conditions

Figure 5.26: Receiver estimated velocities using GPS constellation for a 15 minute interval: addressing traveling ionospheric disturbances a data snooping technique. VADASE solutions are displayed on right-hand y axis. Code Minus Carrier observation for L1 carrier phase is displayed on the left-hand y axis. Data are obtained from Spirent simulator using the scenario described in table 5.15

5.5.3 Tropospheric front

For more than 20 years, GPS has been widely used as a reliable and rather cheap technique (especially compared to radiosondes) to estimate the water vapor content in the troposphere. In fact, this quantity is highly and rapidly variable both in the time and spatial scale. In addition, due to the complexity of the mechanisms that control the chemical reactions taking place in the atmosphere, it is still not possible to effectively model or predict the water vapor distribution.

For what attains to the research presented in this work, it is important to recall that rapid changes in the water vapor distribution can cause a rapid range variation for the pseudoranges and carrier phase observations of GNSS satellites.

The limitations of the technique, due to delays in the implementation procedure at the moment of this writing, were already discussed in 5.5.1

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9The limitations of the technique, due to delays in the implementation procedure at the moment of this writing, were already discussed in 5.5.1
5.5. Addressing rapid and sharp changes in atmospheric conditions

As it was explained for the TID, this phenomenon can be thought of as a front of tropospheric perturbation moving with defined characteristics and disturbing satellites signals. To prove the variometric algorithm robustness, these conditions were recreated in a new simulation scenario by means of a User command file. In details, the structure of the file is similar to the one explained in section 5.5.2 but for the fact that the disturbance due to the tropospheric front acts as a delay both for pseudoranges and carrier phases observations. The general parameters used to simulate this new scenario are summarized in table 5.16.

<table>
<thead>
<tr>
<th>SCENARIO PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date — Rate</td>
</tr>
<tr>
<td>Position</td>
</tr>
<tr>
<td>Tropospheric delay</td>
</tr>
<tr>
<td>Ionospheric delay</td>
</tr>
</tbody>
</table>

Table 5.16: Scenario 8: tropospheric front

Uploading a tuned User command file to Spirent simulator the observations coming from GPS satellites G14, G16, G31 were altered imposing an additional delay due to the rapid changes of the tropospheric conditions. The delay was simulated to impact the mentioned satellites at different epochs (i.e. initially only G31, then G14 and G16 at the same time). The least squares estimation of the 3D velocity was based upon the entire set of equations 5.8 that could be written (separately for L1 and L2) for each satellite in common to two generic consecutive epochs \((t, t + 1)\). The estimated receiver velocities are shown, with respect to the right-hand y axis, in figure 5.27.

The impact of the tropospheric front displayed as small jumps in the estimated receiver velocities. The first little jump is notable around epoch 88100 [sec of week, GPS time] and corresponds to the ramping tropospheric delay for satellite G31. A major jump is clearly shown around epoch 88300 [sec of week, GPS time] and is due to the combined effect of the G14 and G16 delays. Here, it is important to underline that the high acquisition rate tends to reduce the blow of the disturbing effect of the tropospheric ramp. In fact, the jumps in the estimated velocities are rather small, the first one being barely visible. Figure 5.28 shows the estimated receiver displacements obtained processing the same data with a reduced observations rate of 1 Hz.

The comparison of figures 5.27 and 5.28 clearly shows how the impact of the same phenomenon maps on a different extent on the estimated receiver displacements according to the observations acquisition rate. This aspect might be crucial and must be taken into account when implementing a filtering technique.
5.5. Addressing rapid and sharp changes in atmospheric conditions

Figure 5.27: Receiver estimated velocities using GPS constellation for a 15 minute interval. Data are obtained from Spirent simulator using the scenario described in table 5.16

in order to detect the observations that suffer from disturbances and to remove them before solving for the receiver displacements. These and other understandings have led to designing a data snooping technique whose guiding parameters are dependent on the observation acquisition rate (e.g. the higher the acquisition rate the lower the threshold value that will be used to trigger the data snooping, see 5.5.1).

Figure 5.29 shows the results of the data snooping technique implemented in VADASE software and applied to the 20 Hz acquisition rate (the higher the rate the more difficult for the snooping technique to choose the correct observation to be removed) and the tropospheric ramp.

The large number of observations that are removed from the solutions depends on the low value of the threshold chosen to trigger the filtering process. Overall, the impact of the ramp is effectively removed.
5.5. Addressing rapid and sharp changes in atmospheric conditions

Figure 5.28: Receiver estimated velocities using GPS constellation for a 15 minute interval: acquisition rate influences on the impact of the tropospheric ramp. Data are obtained from Spirent simulator using the scenario described in table 5.16
5.5. Addressing rapid and sharp changes in atmospheric conditions

Figure 5.29: Receiver estimated velocities using GPS constellation for a 15 minute interval: addressing rapid changes of tropospheric conditions. Data are obtained from Spirent simulator using the scenario described in table 5.16
6.1 Denali earthquake, Alaska, November 3, 2002

The Denali fault earthquake occurred on November 3, 2002 at 22:12:41 Coordinated Universal Time (UTC) and ruptured a total distance of 340 km. As a matter of fact, the seismic energy released by this strong \( M_w = 7.9 \) and shallow (5.0 km) earthquake led to amplified Love and Rayleigh waves which widely propagated to the south-east (see [12] and references therein). Due to these particular features, the large surface waves stemmed from this earthquake caused seismic instruments to clip, even at extensive distances (e.g. greater than 2000 km), without triggering strong-motion instruments.

It was indeed in this occasion that GPS data coming from high-rate (1 Hz) densely distributed networks were used to a large extent to investigate the coseismic displacements and to measure the deformations caused by the earthquake. Several researchers compared the performances of GPS in detecting the waveforms induced by the seismic event with those of more classical seismological instruments (e.g. accelerometers and seismometers) [51, 48]. It was shown that horizontal displacements exceeding 0.015 m amplitude could be revealed at the level of 2-3 mm accuracy by an high-rate real-time GPS network located as far as
3900 km from the epicenter [20]. Since many broadband seismometers saturated, the Denali GPS seismograms were used to study in details the earthquake triggering mechanism [36]. Finally, several GPS stations throughout North America were analyzed in order to derive displacement waveforms for this event and to assess the impact that instrumentation and error reduction strategies could have over the noise characteristics of displacement time series [12].

The variometric approach was applied to high-rate (1 Hz) observations of BREW GPS station, which is located about 2400 km away from the seismic source. In fact, this choice responds to the issue of testing the innovative approach developed in this work at large distances. However, it has to be underlined that BREW was well located with respect to the maximum earthquake propagation direction (i.e. south-east): the analysis of data coming from GPS sites located at comparable distances but perpendicularly to the propagation direction showed no large displacements. Figure 6.1 shows the station position with respect to the earthquake source.

The observations for the 15 minutes interval from 22:15:00 to 22:30:00 November 3, 2002, GPS time for BREW station were downloaded from IGS high-rate (1 Hz) network in the form of RINEX file (i.e. file name brew307w15.02o). As figure 6.2 shows, the seismic signature was indeed evident from the receiver 3D velocities estimated for the whole interval. In particular, it is worth noting how the receiver underwent significant displacements even in the vertical component, in which, generally, the seismic signature is drowned by the noise level (e.g. stations P744 and P494 Baja, California earthquake described in section 6.3). Moreover, the solutions for the vertical component are affected by quite a large bias (0.002 m). This effect, most probably due to mismodeling in the variometric model, will display as a drift when the estimated velocities will be integrated to obtain the receiver waveforms.

At this point, to obtain the receiver waveforms in a global reference frame, the estimated velocities were integrated over the interval when the earthquake occurred. Here, looking at the estimated velocities, the receiver displacements were practically exhausted within the 5 minutes interval from 22:22:00 to 22:27:00, which, therefore, was chosen as the integration interval. Figure 6.3 shows the receiver waveforms and coseismic displacements for the mentioned interval.

For the sake of comparison with the solutions obtained by other research groups, it is possible, limiting to the East component, to face figure 6.3 with figure 11 reported in [12], who adopted the instantaneous positioning strategy [19] to analyze BREW GPS data. Additionally, the waveforms were computed Along (A) the and Cross (C) to propagation direction in order to compare them with solutions showed in [48, figures 3 and 4] and obtained applying PPP approach. Figure 6.4 reports the rotated waveforms.
Figure 6.1: Denali earthquake - Map of seismic source and station position
Figure 6.2: BREW - estimated 3D velocities using GPS broadcast products (orbits and clocks) in the 15 minutes interval from 22:15:00 to 22:30:00 on November 3, 2002, GPS time

From the waveforms shown in figures 6.4 and 6.3 it is possible to see that seismic waves caused a maximum peak to peak receiver oscillation of $\sim 0.200$ m in the horizontal components.

The coseismic displacements at the end of the 5 minute interval are almost negligible in the East and North components whereas the Up results in a final displacement of 0.073 m. Nonetheless, this apparently large value could be explained recalling the large positive bias that affected the estimated velocities. In fact, this bias caused the integrated velocity to drift up reaching approximately 0.090 m when the vertical displacement started (epoch 80690). This consideration, driven by the analysis of the estimated receiver velocities, takes the coseismic displacement for the Up component back to a value of approximately -0.020 m.

Figure 6.3: BREW - waveforms and coseismic displacements using GPS broadcast products (orbits and clocks) in the 300 seconds interval from 22:22:00 to 22:27:00 on November 3, 2002, GPS time. A direct comparison, for the East component, is possible with figure 11 reported in [12]

Figure 6.4: BREW - waveforms and coseismic displacements using GPS broadcast products (orbits and clocks) - Along and perpendicular (C) to the waves propagation direction - in the 300 seconds interval from 22:22:00 to 22:27:00 on November 3, 2002, GPS time. A direct comparison is possible with figures 3 and 4 reported in [48]
6.2 L’Aquila earthquake, Italy, April 6, 2009

On April 6, 2009, at 01:32:39 UTC an earthquake of magnitude ($M_w$) 6.3 hit L’Aquila city (central part of Italy) and the nearby towns. L’Aquila is located within the Appennines: a mountain belt that traverses the Italian country from the Po basin (North) down to the Gulf of Taranto. Due to its complex geology, this region expresses many different tectonic styles and, in the recent past, was interested by several significant earthquakes (e.g. in 1997 a $M_w$ 6.0 earthquake 85 km north-northwest of the April 6, 2009, event destroyed approximately 80000 homes in the Marche and Umbria regions causing 11 casualties [79]). Nonetheless, L’aquila city was not structurally prepared to withstand the earthquake and, as a consequence, almost 300 people died and most of the buildings suffered so severe damages that they result still completely unavailable at the moment of this writing.

This earthquake occurred on the Paganica fault and was recorded by a number of 1 Hz GPS stations belonging to the Rete Integrata Nazionale GPS (RING). In addition, a few survey style GPS benchmarks were also installed a few days before the main shock [5] In particular, at CADO (Fossa) GPS site a Leica
AX1202 antenna was installed on a 1.5 m tall concrete pillar and the receiver (Leica GX1230) was set up with a 10 Hz sampling interval. No GPS sites outside the epicentral area were recording with the same sampling rate [7]. Figure 6.5 shows the station position with respect to the earthquake source.

Using PPP analysis the waveforms and the coseismic displacements caused by L’Aquila earthquake on the mentioned sites were retrieved and compared with the closest solutions supplied by strong motion instruments (e.g. accelerometers) [7]. In the framework of a cooperation with Istituto Nazionale di Geofisica e Vulcanologia (INGV), the high-rate (10 Hz) GPS data of CADO station were processed using the variometric approach. Figure 6.6 displays the earthquake signature in the 1 minute interval from 01:32:30 to 01:33:30 on April 6, 2009, GPS time.

![Figure 6.6: CADO - estimated 3D velocities using GPS broadcast products (orbits and clocks) in the 60 s interval from 01:32:30 to 01:33:30 on April 6, 2009, GPS time](image)

To retrieve the receiver waveforms, those velocities were integrated, within the selected interval, and are displayed in figure 6.7. As it was noted in [7], CADO shows high amplitude displacements that result *in-phase* for the East and North component (the maximum peak-to-peak displacement resulted just over 0.400 m, in the East direction). For the Up component a displacement of 0.200 m is
Figure 6.7: CADO - waveforms and coseismic displacements using GPS broadcast products (orbits and clocks) in the 60 s interval from 01:32:30 to 01:33:30 on April 6, 2009, GPS time

The results achieved by the variometric approach are extremely accurate in describing the seismic waveforms. On the other hand, it is quite evident the presence of a drift in the coseismic displacements (especially in the North and Up components). This effect, which is due to residual biases in the estimated velocities, should be considered carefully if one wants to duly compute the coseismic displacements.

6.3 Baja earthquake, California, April 4, 2010

A magnitude $M_w$ 7.2 earthquake occurred at 22:40:42 UTC, on Sunday, April 4, 2010, Day Of the Year (DOY) 094, 63 km SSE of Calexico, CA, in the upper Baja California peninsula of Mexico. Tremors lasted 40 seconds causing extensive damage to buildings in the area. This earthquake has been named the El Mayor-Cucapah earthquake [84].
University NAVSTAR Consortium (UNAVCO) is a non-profit University Governed consortium which aims to facilitate geoscience, research and education using geodesy. This consortium fosters communication for an by scientists and educator and, generally speaking, undertakes many activities to support the scientific community. In this framework, UNAVCO coordinates data exchanges and organizes web-spaces where discussions and comparisons of results stemming from the analysis either of earthquakes or of other notable events can take place. In particular, for the El Mayor-Cucapah event UNAVCO downloaded and archived 5 Hz data from all PBO GPS stations with 97% data completeness within 200 km of the northern end of the April 4, 2010 El Mayor-Cucapah earthquake rupture. The observations were collected in standard RINEX files and made publicly available to the users through File Transfer Protocol (ftp) service. Additionally, it was set up an event response forum [84] in which the scientific community was encouraged to report any interesting result concerning the earthquake.

Taking advantage from the high-rate data availability, some of the PBO stations were processed using the variometric approach implemented in VADASE software. In particular, it was decided to process three stations (i.e. P494, P496, and P744) for the sake of comparison with the results obtained, employing different strategies, by other research groups and shown in the event response forum. Figure 6.8 shows the map with the three selected stations and the seismic source.

It is worth to underline that all the stations were processed separately (i.e. single station approach) using the GPS broadcast products (orbits and clocks), which are available in real-time. The complete variometric model, previously described in equation 4.7, was employed in the least squares solution for the 3D receiver velocities $\Delta \xi_r$. Then, to obtain the receiver waveforms in a global reference frame, the estimated velocities were integrated over the interval when the earthquake occurred.

6.3.1 Station P494

The earthquake signature resulted clearly evident in the 1 hour interval time series of the estimated receiver velocities (figure 6.9).

The effectiveness of the variometric approach in estimating the real-time 3D velocities due to an earthquake is further shown with more details when zooming in the 220 seconds interval from 22:40:20 to 22:44:00, as it is highlighted in figure 6.10.

Looking at the planimetric components of the receiver velocities the triggering instant when the receiver underwent the seismic motion could be eventually identified. Additionally, it is interesting to note how the seismic signal drowned the small spikes characterizing the receiver velocities before and after the event.
Figure 6.8: Baja earthquake - Map of seismic source and stations position
These spikes, which result from the observations noise, are displayed only before and after the interval when the earthquake occurred. On the other hand, solutions for the vertical component resulted less accurate (approximately by a factor of 2) and the earthquake signature was barely visible in the time series of the Up direction. However, concerning the positioning accuracy, the looseness of the vertical component with respect to the planimetric ones is a very well-known systematic effect that characterizes GNSS systems.

Figure 6.11 shows the receiver waveforms obtained by integrating the estimated 3D velocities in the same interval of 220 seconds.

At a first glance it appears that for the vertical component the effect of the high noise is predominant over the waveforms induced by the earthquake. On the other hand, the East and North components neatly display the receiver oscillations. At this point, to assess the waveforms retrieved with the variometric approach it has been chosen to assume a reference solution for station P494. Exploiting the event response web site set up by UNAVCO, the solution (reported in figure 6 at [84]) obtained by Yehuda Bock at UCSD and Sharon Kedar at JPL using the instantaneous positioning approach was chosen as reference. Hence, figure 6.12 displays the planimetric components for the 200 seconds interval for
The visual comparison with the reference solution clearly shows the efficacy of the variometric approach in retrieving the waveforms induced by the earthquake. Further, as regards the maximum displacements underwent by the receiver (i.e. the maxima and minima peaks in the receiver oscillations), the agreement between the two solutions is at the centimeter level. This latter achievement is rather significant since the maximum displacement caused by an earthquake is a crucial parameter for estimating the earthquake magnitude ($M_w$).

A final comment about the coseismic displacement obtained with VADASE should be made. To this aim, it is worth to remind that the waveforms achievable with the variometric approach stem from the integration of the estimated receiver waveforms and, therefore, are very sensitive to estimation biases that cumulate over time and display their effect as a drift. Here, the East component does not show any coseismic displacement for the interval analyzed; this is in agreement with the reference solution. On the other hand, for the North component the coseismic displacement at the end of the interval reaches $-0.25$ m. whereas the reference solution settles at a value of $-0.20$ m. Nonetheless, this difference

Figure 6.10: P494 - estimated 3D velocities using GPS broadcast products (orbits and clocks) in the 220 s interval 22:40:20-22:44:00 on April 4, 2010, GPS time
6.3. Baja earthquake, California, April 4, 2010

Figure 6.11: P494 - waveforms and coseismic displacements using GPS broadcast products (orbits and clocks) in the 220 s interval 22:40:20-22:44:00 on April 4, 2010, GPS time

could be eventually explained by the slight drift that affects the North component achieved with VADASE (in figure 6.12, mostly after epoch 81750 [sec of week], 22:42:30) and that results in an additional displacement of \( \sim 0.05 \) cm in the final 100 seconds of the analyzed interval.

6.3.2 Station P496

GPS permanent station \( P496 \) was slightly closer to the earthquake source (i.e. \( \sim 62 \) km) with respect to station \( P494 \). To estimate the receiver 3D velocities using the variometric approach the GPS observations stored in the RINEX file \( p496094w.10o \) at 5 Hz acquisition rate were analyzed. As it was stated for the analysis of permanent station \( P494 \), the earthquake signature resulted evident in the 1 hour interval time series of the receiver velocities. In figure 6.13 the results are shown selecting the 200 seconds interval from 22:40:40 to 22:44:00, GPS time.

Again, the triggering instant when the receiver underwent the seismic motion could be identified looking at the planimetric components of the estimated velocities. The vertical component showed the same behavior already noted for
6.3. Baja earthquake, California, April 4, 2010

Figure 6.12: P494 coseismic displacements obtained by VADASE using broadcast products; 200 s interval from 22:40:40 - 22:44:00 on April 4, 2010, GPS time. A direct comparison with the waveforms and coseismic displacements recovered by Yehuda Bock at UCSD and Sharon Kedar at JPL (UNAVCO)
6.3. Baja earthquake, California, April 4, 2010

Figure 6.13: P496 - estimated 3D velocities using GPS broadcast products (orbits and clocks) in the 200 s interval 22:40:40-22:44:00 on April 4, 2010, GPS time

station P494 and the earthquake signature resulted rather drowned in the estimation noise. Figure 6.14 shows the receiver waveforms obtained integrating the estimated 3D velocities in the interval selected for the analysis.

For the sake of comparing the waveforms with those achieved with other strategies, it has been chosen to take as reference the solution achieved using PPP by Kristine Larson (University of Boulder, Colorado) and available at the UNAVCO event response web site. Hence, figure 6.12 can be directly compared with PPP solution available at [84, fig. 5]. For the sake of completeness, it has been chosen to display the 3D waveform solutions even though the vertical component is not provided in the reference results.

As it is notable by the visual comparison with the reference, VADASE approach is very effective in retrieving the receiver waveforms induced by the earthquake. Additionally, the oscillations peaks displayed by the two solutions agree at the centimeter level. Coming to the coseismic displacement, it is important to note that variometric solutions are influenced by a slight drift in the planimetric components. As regards the East, the effect is evident before the earthquake and yields to a coseismic displacement of 0.01 m, which is in agreement with the few centimeters displayed in the reference. As it was the case for station P494,
6.3. Baja earthquake, California, April 4, 2010

Figure 6.14: P496 - waveforms and coseismic displacements using GPS broadcast products (orbits and clocks) in the 200 s interval 22:40:40-22:44:00 on April 4, 2010, GPS time

the North component displays a major drift with respect to the East one (in figure 6.15, mostly after epoch 81750 [sec of week], 22:42:30, GPS time). Here, VADASE coseismic displacement reaches $-0.27 \text{ m}$ whereas the reference settles by just under $-0.20 \text{ m}$. Again, the difference between the two approaches can be mostly explained by the drift in the variometric solution (i.e. North coseismic displacement, which was $-0.21 \text{ m}$ at epoch 81750, increased by just over 6 centimeters in the final 100 seconds interval due to the effect of the drift).

6.3.3 Station P744

Station P744 is located 67 km a part from the earthquake source. The 3D receiver velocities were estimated with the vatiometric approach using the observations available in the RINEX file $p744094w.10o$ at 5 Hz acquisition rate. Figure 6.16 shows the results for the 200 seconds interval from 22:40:40 to 22:44:00, GPS time.

The vertical component appeared to be unaffected by the earthquake signature. On the contrary, the planimetric components neatly displayed the trigger-
6.3. Baja earthquake, California, April 4, 2010

Figure 6.15: P496 - waveforms and coseismic displacements obtained by VADASE using broadcast products; 200 s interval from 22:40:40 - 22:44:00 on April 4, 2010, GPS time. A direct comparison is possible with the waveforms and coseismic displacements recovered by Kristine Larson, University of Boulder, Colorado. (UNAVCO - P496 - Figure 5)
6.3. Baja earthquake, California, April 4, 2010

Figure 6.16: P744 - estimated 3D velocities using GPS broadcast products (orbits and clocks) in the 200 s interval 22:40:40-22:44:00 on April 4, 2010, GPS time

As it was for station P496, PPP solution obtained by Kristine Larson (University of Boulder, Colorado) and available at the UNAVCO event response website was chosen as reference to compare the effectiveness of the variometric algorithm. Hence, figure 6.18 can be directly compared with PPP solution available at [84, fig. 5]. For the sake of completeness, it has been chosen to display the 3D waveform solutions even though the vertical component is not provided in the reference results.

From the visual comparison between the two solutions, some comments, similar to those already stated for station P496, can be formulated. Both approaches definitely agree on the maximum displacements and waveforms underwent by the receiver due to the earthquake. Again, the planimetric components displayed a slight drift which caused the coseismic displacements to slightly differ from those obtained by the reference. For the East component the difference is rather unimportant (i.e. no coseismic displacement in the variometric approach with respect to the very few centimeters displayed by the reference); as regards the North, the
6.3. Baja earthquake, California, April 4, 2010

Figure 6.17: P744 - waveforms and coseismic displacements using GPS broadcast products (orbits and clocks) in the 200 s interval 22:40:40-22:44:00 on April 4, 2010, GPS time

difference between the solutions amounts to several centimeters. This difference almost cancels out not accounting for the drift in the final 100 seconds of the variometric approach (i.e. the drift caused the variometric displacement to grow approximately by just over 8 centimeters in the final part of the interval, from the -0.05 m at epoch 81750 to the -0.14 m at epoch 81640).
6.3. Baja earthquake, California, April 4, 2010

Figure 6.18: P744 - waveforms and coseismic displacements obtained by VADASE using broadcast products; 200 s interval from 22:40.40 - 22:44:00 on April 4, 2010, GPS time. A direct comparison is possible with the waveforms and coseismic displacements recovered by Kristine Larson, University of Boulder, Colorado. (UNAVCO - P744 - Figure 5)
6.4 The great Tohoku-oki earthquake, Japan, March 11, 2011

On Friday, March 11, 2011, at 05:46:24 UTC a magnitude $M_w$ 9.0 earthquake hit the Island of Honshu (Japan) generating a tremendous tsunami that caused a large number of victims, wide damages and nuclear fear induced by the severe accident occurred to the Fukushima nuclear plant. The majority of casualties occurred in Iwate, Miyagi and Fukushima from a Pacific-wide tsunami with a maximum runup height of 37.88 m at Miyako [80]. This great earthquake was preceded by a series of large foreshocks over the previous two days, beginning on March 9 with an $M_w$ 7.2 event approximately 40 km a part from the epicenter, and continuing with many aftershocks with a magnitude greater than 6.

Even though in this case nothing or very little could have been done to avoid the dramatic number of victims, this event raised once more the attention towards the importance of deploying tsunami early warning systems and related real-time processing strategies for analyzing geophysical and geodetic data.

It was for this tremendous occurrence that the variometric approach showed its potentials to obtain waveforms and coseismic displacements caused by an earthquake in real-time and using a single receiver. With a few hours of latency with respect to the event\(^1\), high-rate (1 Hz) observations of the IGS stations MIZU and USUD (map in figure 6.19) were downloaded from the web in the form of RINEX files and processed using VADASE software. The results obtained for the receiver waveforms and coseismic displacements caused by the Tohoku-oki earthquake were the first to be announced among the scientific community and were spread by means of the IGS mailing list [22].

It is worth underlying once more that, from a practical point of view, all the necessary data used by the variometric approach (i.e. observations and broadcast products) were available directly in real-time at the moment the event occurred. Further, the computational burden needed to solve for the receiver velocities was insignificant (data of both the aforementioned stations were processed in less than thirty seconds). In fact, this can be definitely satisfied by modern receivers firmware capabilities. This is to state that, at present, there are no constraining factors with respect to the implementation of the variometric algorithm in the GNSS receiver firmware. This would constitute a monitoring device able to detect in real-time the receiver displacements. A simple transmission equipment

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\(^1\)The earthquake occurred at 05:46 UTC. The data were processed about 5 hours later. This time span was necessary to reach the working place, at “Sapienza” University of Rome, Italy, and collect the data from the IGS web site. The processing time for an interval of 15 minutes was insignificant (less than thirty seconds) .
Figure 6.19: Japan earthquake - Map of seismic source and stations position
could be added to the receiver in order to communicate with a remote monitoring system. Imaging such a configuration, in case an earthquake occurs, the waveforms could be retrieved in real-time, with the centimeter level accuracy already demonstrated for Baja earthquake. Then, the waveforms could be transmitted to any remote control center that would be able to evaluate whether to raise a tsunami warning or not.

For the Tohoku-oki earthquake, three GPS permanent stations were analyzed: MIZU, USUD and JA01. While the latter station is part of the high-rate network managed by DLR, the former ones are included in the high-rate IGS network. Figure 6.19 shows a map with the location of the selected stations together with the seismic source. In this regard, it is worth to explain how and why the three stations were selected.

To support the scientific community in developing new approaches exploiting the availability of high-rate GPS data, IGS decided to increase the observation rate of a bunch of permanent stations encompassed in its global network to 1 Hz. In addition, these observation are timely formatted in RINEX files that are made available to the users with a latency of $\sim 15$ minutes [43]. In order to monitor the current status of this high-rate global network an automatic tool has been developed at Geodesy and Geomatics Area, Department of Environmental and Civil Engineering, “Sapienza” University of Rome. Given the time instant and the location where an earthquake occurred, this tool takes care of accessing the high-rate IGS data repository and downloading the RINEX observation files for all the usable stations. Then, it displays a map showing the earthquake location and its distance with respect to the closer stations. Applying this tool to scan the IGS network status for the Tohoku-oki earthquake, MIZU and USUD resulted to be the closer usable stations and were processed immediately after (see page 124) the event. JA01 station is managed by DLR and its observations were kindly provided to the scientific community [4] by Dr. Thomas Dautermann few days after the event.

It is worth to underline that all the stations were processed separately (i.e. single station approach) using the GPS broadcast products (orbits and clocks) which are available in real-time. The complete variometric model, previously described in equation 4.7, was employed in the least squares solution for the 3D receiver velocities $\Delta \xi_r$. Then, to obtain the receiver waveforms in a global reference frame, the estimated velocities were integrated over the interval when the earthquake occurred.
6.4.1 Coseismic displacements

Figures 6.20, 6.21 and 6.22 show the coseismic displacements achieved by the variometric approach for USUD, MIZU and JA01 stations, respectively. Overall, the time interval picked up for the earthquake analysis is 240 seconds long. However, since the stations distances from the earthquake are different, MIZU suffered the displacements caused by the seismic wave approximately 1 minute in advance with respect to the others. Hence, the intervals are not the same for the three stations.

![Waveforms and coseismic displacements](image)

Figure 6.20: MIZU - waveforms and coseismic displacements for the 240 seconds interval from 05:46:10 to 05:50:10, March 11, 2011, DOY 070, GPS time, obtained with the variometric approach using RINEX observations file mizu070f45.11o (1 Hz acquisition rate) and RINEX broadcast ephemerides brdc0700.11n

The coseismic displacements achieved by VADASE for MIZU station at the end of the 4 minutes interval from 05:46:10 to 05:50:10, GPS time, are 2.06 m in the East component, -1.17 m for the North component and 0.20 m for the vertical one. The waveforms can be compared with the solutions reported in [4].

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6.4. The great Tohoku-oki earthquake, Japan, March 11, 2011

Figure 6.21: USUD - waveforms and coseismic displacements for the 240 seconds interval from 05:47:00 to 05:51:00, March 11, 2011, DOY 070, GPS time, obtained with the variometric approach using RINEX observations file `usud070f45.11o` (1 Hz acquisition rate) and RINEX broadcast ephemerides `brdc0700.11n`
6.4. The great Tohoku-oki earthquake, Japan, March 11, 2011

*USUD* and *JA01* stations, whose data were analyzed within the same 240 seconds time interval from 05:47:00 to 05:51:00, GPS time, displayed significantly lower coseismic displacements, which are shown in table 6.1

![Waveforms and coseismic displacements](image)

Figure 6.22: JA01 - waveforms and coseismic displacements for the 240 seconds interval from 05:47:00 to 05:51:00, March 11, 2011, DOY 070, GPS time, obtained with the variometric approach using RINEX observations file *ja01070f.11o* (20 Hz acquisition rate) and RINEX broadcast ephemerides *brdc0700.11n*

6.4.2 Comparison with Precise Point Positioning approach of National Resources of Canada

The tremendous Tohoku-oki earthquake of March 11, 2011 was the second highest magnitude earthquake of the last twenty years [81]. Most of the casualties were caused by the tsunami that was triggered by the earthquake and that devastated Japan coastlines. Immediately after the event, the Group on Earth Observations (GEO), in partnership with many other organizations and research institutes, set
6.4. The great Tohoku-oki earthquake, Japan, March 11, 2011

Table 6.1: JA01 and USUD - coseismic displacement for the 240 seconds interval from 05:47:00 to 05:51:00, March 11, 2011, DOY 070, GPS time

<table>
<thead>
<tr>
<th>Time Period</th>
<th>East [m]</th>
<th>North [m]</th>
<th>Up [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>05:47:00 - 05:51:00</td>
<td>0.09</td>
<td>-0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>JA01</td>
<td>0.17</td>
<td>-0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>USUD</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

up the Tohoku-oki event response website [4] in order to collect all the responses of the scientific community and to facilitate the understanding of the tectonic processes interesting Japan.

Concerning the contribution supplied by GNSS data analysis, the first results of the waveforms and coseismic displacements caused by the earthquake were provided by the variometric approach presented in this work. Later on, the mentioned website was enriched with solutions produced by several research groups employing different processing strategies.

Given the outstanding performance of VADASE algorithm (i.e. being the first to provide coseismic displacement solutions), more interest arose towards the accurate comparison of the variometric solutions with those furnished by other methods which represent the state of the art of the GPS seismology (i.e. the instantaneous positioning [19] and the Precise Point Positioning [85]). In particular, the results obtained for the three stations of MIZU, USUD and JA01 and described in section 6.4.1, were compared with those stemming from the PPP approach implemented in the software developed at NRCan [62]. This additional work was made possible thanks to the precious support of Dr. Henton Joe and Dr. Dragert Herb, NRCan, who computed the PPP solutions for the aforementioned stations.

At the beginning, for each station, the two solutions were aligned (i.e. their difference was set to zero) at a conventional initial epoch. This time tag varied according to the different distances from the earthquake and, as a matter of fact, was decided to be the initial epoch of the 4 minutes intervals when the stations were analyzed (see section 6.4.1). Then, the estimated positions were differenced at the corresponding epochs and the agreement between the solutions was evaluated by means of the RMSE\(^2\) of the differences\(^3\). Additionally, two other aspects were investigated:

1. the overall 4 minutes interval was split in 4 parts (each part 1 minute long)

\[ \text{RMSE} = \sqrt{\text{bias}^2 + \text{st.dev.}^2} \]

\[ \text{For the generic epoch } i \text{ the difference is given as: } \text{DIFF}_i = \text{VADASE}_i - \text{PPP}_i \]
in order to inspect the agreement reliance with the earthquake duration. This was interesting to explore how possible mismodeling errors in the variometric algorithm might impact the recovered waveforms.

2. minima and maxima displacements, obtained by the two strategies, were visually identified considering, separately, the vertical and the horizontal components. The differences between these values yield to the maximum displacement suffered by the receiver. This is an important parameter to estimate the earthquake magnitude ($M_w$) and, consequently, to eventually raise a tsunami warning. In fact, peak ground motion distributions provide information that may be used to assess earthquake impacts [67]. In such sense, the two solutions were compared.

**MIZU**

Figures 6.23, 6.24 and 6.25 show the results of the comparison between VADASE and NRCan PPP for the East, North and Up components, respectively, of MIZU station. The waveforms produced with the variometric approach are displayed in red, whereas the Precise Point Positioning ones are displayed in blue. The black line reports the trend of the differences between VADASE and NRCan. In order to assess the maximum displacements suffered by the receiver according to the two solutions, 12 values (corresponding to relative minima and maxima in the solutions) were identified, separately, for the horizontal and the vertical components. The difference between these consecutive values was taken as an index, for each solution, of the magnitude of the movements caused by the earthquake. In the figure, minima and maxima relative to each solution are evidenced by means of black circles.

At the end of the 4 minutes interval from 05:46:10 to 05:50:10 the difference in the displacements obtained by the two solutions was -0.011 m for the horizontal components and 0.324 m for the vertical one. Overall, the RMSE of the differences was 0.025 m in East, 0.014 m in North and 0.219 m in the Up component.

Table 6.2 reports the results of the statistics and shows how the agreement between the solutions changed with earthquake duration. As regards the horizontal components, the RMSE of the differences remained rather constant while the interval length increased from 1 to 4 minutes. On the contrary, for the vertical component the increasing length caused a significant degradation in the solutions agreement. In more details, the standard deviation stayed constant whereas the bias grew up, approximately by 0.050 m for each minute, reaching 0.196 m when the whole 4 minutes were analyzed.

For MIZU station, VADASE and NRCan PPP softwares definitely agreed in quantifying the maximum displacements suffered by the receiver. In fact, the
6.4. The great Tohoku-oki earthquake, Japan, March 11, 2011

Figure 6.23: MIZU - VADASE and PPP comparison on waveforms and coseismic displacement for the East component for the 240 seconds interval from 05:46:10 to 05:50:10, March 11, 2011, DOY 070, GPS time
Figure 6.24: MIZU - VADASE and PPP comparison on waveforms and coseismic displacement for the North component for the 240 seconds interval from 05:46:10 to 05:50:10, March 11, 2011, DOY 070, GPS time
Figure 6.25: MIZU - VADASE and PPP comparison on waveforms and coseismic displacement for the Up component for the 240 seconds interval from 05:46:10 to 05:50:10, March 11, 2011, DOY 070, GPS time
Table 6.2: MIZU - Statistics of VADASE and PPP comparison on waveforms and coseismic displacements for the 240 seconds interval from 05:46:10 to 05:50:10, March 11, 2011, DOY 070, GPS time

<table>
<thead>
<tr>
<th>MIZU</th>
<th>Interval</th>
<th>st.dev. [m]</th>
<th>bias [m]</th>
<th>RMSE [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>EAST</td>
<td>05:46:10</td>
<td>0.020</td>
<td>-0.022</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>0.016</td>
<td>-0.025</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>0.014</td>
<td>-0.025</td>
<td>0.029</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>0.016</td>
<td>-0.020</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>0.016</td>
<td>-0.020</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Δ (MIN-MAX)</td>
<td>0.010</td>
<td>0.002</td>
<td>0.010</td>
</tr>
<tr>
<td>NORTH</td>
<td>05:46:10</td>
<td>0.004</td>
<td>-0.004</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>0.009</td>
<td>-0.012</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>0.010</td>
<td>-0.008</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>0.010</td>
<td>-0.010</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>Δ (MIN-MAX)</td>
<td>0.012</td>
<td>-0.001</td>
<td>0.012</td>
</tr>
<tr>
<td>UP</td>
<td>05:46:10</td>
<td>0.038</td>
<td>0.052</td>
<td>0.065</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>0.075</td>
<td>0.119</td>
<td>0.140</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>0.086</td>
<td>0.160</td>
<td>0.182</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>0.097</td>
<td>0.196</td>
<td>0.219</td>
</tr>
<tr>
<td></td>
<td>Δ (MIN-MAX)</td>
<td>0.024</td>
<td>0.017</td>
<td>0.030</td>
</tr>
</tbody>
</table>
difference resulted in $\sim 0.010$ m for the horizontal components and $0.030$ m for the vertical one.

Finally, the correlation coefficient between the two solutions reached 99.9% for East and North components and laid around 65.0% for the Up.

USUD

Figures 6.26, 6.27 and 6.28 show the results of the comparison between VADASE and NRCan PPP for the East, North and Up directions, respectively, of USUD station. The waveforms produced with the variometric approach are displayed in red, whereas the Precise Point Positioning ones are displayed in blue. The black line reports the trend of the differences between VADASE and NRCan. Further, minima and maxima relative to each solution are evidenced by means of black circles.

Figure 6.26: USUD - VADASE and PPP comparison on waveforms and coseismic displacement for the East component for the 240 seconds interval from 05:47:00 to 05:51:00, March 11, 2011, DOY 070, GPS time
Figure 6.27: USUD - VADASE and PPP comparison on waveforms and coseismic displacement for the North component for the 240 seconds interval from 05:47:00 to 05:51:00, March 11, 2011, DOY 070, GPS time
6.4. The great Tohoku-oki earthquake, Japan, March 11, 2011

Figure 6.28: USUD - VADASE and PPP comparison on waveforms and coseismic displacement for the Up component for the 240 seconds interval from 05:47:00 to 05:51:00, March 11, 2011, DOY 070, GPS time
At the end of the 4 minutes interval from 05:47:00 to 05:51:00 the difference in the displacements obtained by the two solutions was -0.011 m for the East, -0.068 m for the North and 0.041 m in the Up component. Overall, the RMSE of the differences was 0.021 m in East, 0.027 m in North and 0.158 m in the Up component.

Table 6.3 reports the results of the statistics and shows how the agreement between the solutions changed with earthquake duration. As regards the horizontal components, the RMSE of the differences remained rather constant while the interval length increased from 1 to 4 minutes. On the contrary, the differences in the Up component displayed a bias that resulted significant since the beginning (i.e. when only the first interval of 1 minute length was considered, the bias already amounted to 0.080 m) and that slightly increased with time up to the value of $\sim 0.150$ m.

<table>
<thead>
<tr>
<th>USUD</th>
<th>Interval</th>
<th>st.dev. [m]</th>
<th>bias [m]</th>
<th>RMSE [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>05:46:10</td>
<td>05:47:10</td>
<td>0.011</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>05:48:10</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>05:49:10</td>
<td>0.014</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>05:50:10</td>
<td>0.014</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ MIN-MAX</td>
<td></td>
<td>0.008</td>
<td>0.000</td>
</tr>
<tr>
<td>EAST</td>
<td>05:46:10</td>
<td>05:47:10</td>
<td>0.005</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>05:48:10</td>
<td>0.012</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>05:49:10</td>
<td>0.013</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>05:50:10</td>
<td>0.025</td>
<td>-0.011</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ MIN-MAX</td>
<td></td>
<td>0.010</td>
<td>-0.003</td>
</tr>
<tr>
<td>NORTH</td>
<td>05:46:10</td>
<td>05:47:10</td>
<td>0.042</td>
<td>0.080</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>05:48:10</td>
<td>0.046</td>
<td>0.110</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>05:49:10</td>
<td>0.061</td>
<td>0.143</td>
</tr>
<tr>
<td></td>
<td>05:46:10</td>
<td>05:50:10</td>
<td>0.061</td>
<td>0.146</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ MIN-MAX</td>
<td></td>
<td>0.034</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 6.3: USUD - Statistics of VADASE and PPP comparison on waveforms and coseismic displacements for the 240 seconds interval from 05:47:00 to 05:51:00, March 11, 2011, DOY 070, GPS time

In order to assess the maximum displacements suffered by the receiver according to the two solutions, 12 and 20 values (corresponding to relative minima and maxima in the solutions) were identified for the vertical and the horizontal components, respectively. The difference between these consecutive values was
taken as an index, for each solution, of the magnitude of the movements caused by the earthquake. For *USUD* station, VADASE and NRCan PPP softwares definitely agreed in quantifying the aforementioned indexes. In fact, the difference resulted in $\sim 0.010$ m for the horizontal components and $0.030$ m for the vertical one.

Finally, the correlation coefficient between the two solutions reached 97.5% and 99.8% for the East and North components, respectively, and laid around 84.2% for the Up.

**JA01**

Figures 6.29, 6.30 and 6.31 show the results of the comparison between VADASE and NRCan PPP for the East, North and Up components, respectively, of *JA01* station. The waveforms produced with the variometric approach are displayed in red, whereas the Precise Point Positioning ones are displayed in blue. The black line reports the trend of the differences between VADASE and NRCan. Further, minima and maxima relative to each solution are evidenced by means of black circles.

At the end of the 4 minutes interval from 05:47:00 to 05:51:00 the difference in the displacements obtained by the two solutions was $0.016$ m for the East, $-0.128$ m for the North and $-0.129$ m in the Up component. Overall, the RMSE of the differences was $0.042$ m in East, $0.058$ m in North and $0.053$ m in the Up component.

Table 6.4 reports the results of the statistics and shows how the agreement between the solutions changed with earthquake duration. Here, the RMSE of the differences remained rather constant showing no significant degradation with respect to the increasing time length of the considered interval.

In order to assess the maximum displacements suffered by the receiver according to the two solutions, 13 and 25 values (corresponding to relative minima and maxima in the solutions) were identified for the vertical and the horizontal components, respectively. The difference between these consecutive values was taken as an index, for each solution, of the magnitude of the movements caused by the earthquake. For *JA01* station, VADASE and NRCan PPP softwares definitely agreed in quantifying the aforementioned indexes. In fact, the difference resulted in $\sim 0.005$ m for the horizontal components and $0.010$ m for the vertical one.

Finally, the correlation coefficient between the two solutions reached 99.6% and 95.6% for the East and North components, respectively, and laid around 90.1% for the Up.
6.4. The great Tohoku-oki earthquake, Japan, March 11, 2011

Figure 6.29: JA01 - VADASE and PPP comparison on waveforms and coseismic displacement for the East component for the 240 seconds interval from 05:47:00 to 05:51:00, March 11, 2011, DOY 070, GPS time
6.4. The great Tohoku-oki earthquake, Japan, March 11, 2011

Figure 6.30: JA01 - VADASE and PPP comparison on waveforms and coseismic displacement for the North component for the 240 seconds interval from 05:47:00 to 05:51:00, March 11, 2011, DOY 070, GPS time
Figure 6.31: JA01 - VADASE and PPP comparison on waveforms and coseismic displacement for the Up component for the 240 seconds interval from 05:47:00 to 05:51:00, March 11, 2011, DOY 070, GPS time
## Table 6.4: JA01 - Statistics of VADASE and PPP comparison on waveforms and coseismic displacements for the 240 seconds interval from 05:47:00 to 05:51:00, March 11, 2011, DOY 070, GPS time

<table>
<thead>
<tr>
<th>JA01</th>
<th>Interval</th>
<th>st.dev. [m]</th>
<th>bias [m]</th>
<th>RMSE [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>05:46:10 05:47:10</td>
<td>0.012</td>
<td>0.021</td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td>05:46:10 05:48:10</td>
<td>0.012</td>
<td>0.025</td>
<td>0.028</td>
</tr>
<tr>
<td>EAST</td>
<td>05:46:10 05:49:10</td>
<td>0.020</td>
<td>0.037</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td>05:46:10 05:50:10</td>
<td>0.019</td>
<td>0.037</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td>Δ MIN-MAX</td>
<td>0.005</td>
<td>0.001</td>
<td>0.006</td>
</tr>
<tr>
<td>NORTH</td>
<td>05:46:10 05:47:10</td>
<td>0.007</td>
<td>-0.010</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>05:46:10 05:48:10</td>
<td>0.010</td>
<td>-0.014</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>05:46:10 05:49:10</td>
<td>0.024</td>
<td>-0.029</td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td>05:46:10 05:50:10</td>
<td>0.036</td>
<td>-0.045</td>
<td>0.058</td>
</tr>
<tr>
<td></td>
<td>Δ MIN-MAX</td>
<td>0.006</td>
<td>-0.002</td>
<td>0.006</td>
</tr>
<tr>
<td>UP</td>
<td>05:46:10 05:47:10</td>
<td>0.027</td>
<td>0.054</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td>05:46:10 05:48:10</td>
<td>0.024</td>
<td>0.039</td>
<td>0.046</td>
</tr>
<tr>
<td></td>
<td>05:46:10 05:49:10</td>
<td>0.023</td>
<td>0.041</td>
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</tr>
<tr>
<td></td>
<td>05:46:10 05:50:10</td>
<td>0.045</td>
<td>0.027</td>
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<tr>
<td></td>
<td>Δ MIN-MAX</td>
<td>0.013</td>
<td>0.001</td>
<td>0.013</td>
</tr>
</tbody>
</table>
6.4. The great Tohoku-oki earthquake, Japan, March 11, 2011

Comments

The agreement between VADASE and PPP is clearly dependent on the earthquake duration, mostly for MIZU and USUD stations. The RMSE of the differences between the two solutions ranges from 0.010 ÷ 0.020 m in East and North and 0.060 m in Up, after one minute, to approximately 0.050 m in East and North and (with much larger variability) 0.200 m in Up after four minutes. Here, it is worth to underline (and it is easily notable looking at the former figures) that the dependency of the agreement on the earthquake duration mostly interests the vertical component. In fact, the statistical analysis performed considering each time longer intervals shows that the bias between VADASE and NRCan PPP tends steadily to grow (e.g. for MIZU station this effect is demonstrated by a growth of approximately 0.050 m per minute) causing a parallel growth in the RMSE (i.e. the standard deviation does not vary with the same significance).

On the contrary, the agreement between the two approaches over the maximum displacements appears independent from the earthquake duration. In details, the differences reaches approximately 0.010 m in East and North and 0.030 m in Up.

Finally, the correlation between the solutions is always better than 99% for the planimetric components (but for the East component of USUD and the North component of JA01 which show 97% and 96% correlations, respectively). The vertical component shows slightly lower (and with larger variability) correlations: 65%, 84% and 90% for stations MIZU, USUD and JA01, respectively. Table 6.5 summarizes the correlation results.

<table>
<thead>
<tr>
<th>STATION</th>
<th>CORRELATION [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EAST</td>
</tr>
<tr>
<td>MIZU</td>
<td>99.9</td>
</tr>
<tr>
<td>USUD</td>
<td>97.5</td>
</tr>
<tr>
<td>JA01</td>
<td>99.6</td>
</tr>
</tbody>
</table>

Table 6.5: Correlation statistics of VADASE and PPP comparison for MIZU, USUD and JA01 stations
Chapter 7

Summary and outlook

The present work proposes a brand new approach for estimating, in real-time, the displacements of a single GNSS receiver in the global reference frame. This approach, named Variometric Approach for Displacement Analysis Stand-alone Engine (VADASE), is based upon a so called “variometric” solution that only requires the observations collected by a unique, stand-alone GNSS receiver and the standard GNSS broadcast products (orbits and clocks), which are ancillary information routinely available in real-time as a part of the broadcast satellites navigation message.

In more details, the variometric approach is based upon a rather simple but important understanding: if the data are acquired at high-rate (1 Hz or more), all the effects that interfere with measuring the real geometric distance from the satellite to the receiver (e.g. orbital errors, atmospheric biases) map with little differences in the observations from two consecutive epochs \((t, t + 1)\). Hence, the time single-difference of dual-frequency carrier phase observations eventually displays the changes in the geometric range between the satellite and the receiver. If the receiver remained fixed, the changes in the geometric range only depend on the satellite orbital motion. On the other hand, if the receiver underwent a displacement, this contributes in changing the geometric range.

Overall, VADASE is based upon using the time single-difference of the carrier phase observations collected at high-rate and the standard GNSS products (orbits and clocks) to solve for the 3D receiver displacement between two consecutive epochs \((t, t + 1)\) in real-time. This displacement, if divided by the interval between two consecutive epochs \((t, t + 1)\), is equal to the (mean) velocity over the
interval \((t, t + 1)\) itself. This is, the variometric approach substantially uses the GNSS receiver as a velocimeter.

In order to prove the feasibility of the brand new proposed approach, the variometric algorithm has been implemented in a tuned software, which was appointed with the same name (i.e. VADASE). Exploiting this software to process a large amount of both simulated and real data, the main advantages of VADASE, as well as its limitations, have been investigated in details and are worth being summarized.

To begin with, the high acquisition rate is recognized as a fundamental requirement of the proposed method. In fact, this allows to minimize the impact of the error sources in the time single-difference observation formed between consecutive epochs. This endorses to use broadcast products to compute satellite orbital motion and to adopt simple models to account for the contribution of disturbing effects that vary rather slowly within short time intervals (e.g. Earth tides, ocean loading). As a matter of fact, these latter effects can be even neglected without altering the effectiveness of the algorithm.

In addition, there is no need of data screening in order to address either losses of lock of the signals or cycle slips. In fact, these events can be easily recognized as outliers in the time series of the estimated 3D velocities and they can be thereby removed. Overall, these features result in VADASE being able to work in real-time with no need of ancillary differential information and with very little computational burden (e.g. 1 hour observations at 10 Hz acquisition rate can be processed in less than 1 minute using a standard computer hardware).

On the other hand, the real-time capability of the algorithm, achieved thanks to the described speed and smoothness, is traded off with the limitation of estimating displacements (not positions), between two consecutive epochs. In this regard, provided that continuous data have been acquired, the time series of the estimated 3D velocities can be integrated over a certain interval to retrieve the dynamic movements (waveforms) of the receiver in a global reference frame. However, as an effect of the integration, possible estimation biases that are caused by residuals in modeling the intervening effects (e.g. residual clock errors, orbit errors, atmospheric errors) accumulate over time and result as a drift in the dynamic waveforms obtained for the receiver.

At a first stage, VADASE effectiveness was proven on a simulated example. First, a known displacement (1 cm East and North and 2 cm Up) was synthetically introduced into carrier phase observations collected at 1 Hz rate by the M0SE permanent GPS station (Rome, Italy). Then, these data were processed using the variometric algorithm: the displacements were estimated with an accuracy of \(1 \div 2\) mm in the horizontal and the vertical directions. The solutions obtained
using the GPS broadcast products available in real-time and the best quality products supplied by IGS a posteriori displayed a global agreement of 1 mm for the horizontal components and 2 mm for the height. Given this fundamental proof of success, and provided that the variometric algorithm was originally conceived to contribute in the field of GPS seismology and tsunami warning systems, VADASE was applied to retrieve the coseismic displacements and the waveforms generated by real earthquakes. Most of the significant results were obtained by considering data collected at 1 Hz rate from the IGS station of BREW during the Denali Fault, Alaska earthquake ($M_w$ 7.9, November 3, 2002), at 10 Hz rate from CADO station during the L’Aquila earthquake ($M_w$ 6.3, April 6, 2009), and at 5 Hz rate from some stations included in the UNAVCO-PBO network during the Baja, California earthquake ($M_w$ 7.2, April 4, 2010). All the results were compared with the solutions obtained by other research groups adopting different processing techniques. In all cases, the agreement between VADASE and the other solutions was between few centimeters and a couple of decimeters.

The real-time potentialities of the variometric approach were internationally recognized during the recent tremendous earthquake in Japan ($M_w$ = 9.0, March 11, 2011), when the GNSS research team of “Sapienza” University of Rome was able to provide the first waveforms results among the scientific community. With a few hours latency with respect to the event, the 1 Hz observations of the IGS stations MIZU and USUD were downloaded from the web in the form of RINEX files and processed using VADASE software. Given this important achievement, more interest arose towards the accurate comparison of the variometric solutions with those furnished by other methods which represent the state of the art of the GPS seismology (i.e. the instantaneous positioning [19] and the Precise Point Positioning [85]). In particular, for the mentioned Japanese earthquake, the results obtained for IGS stations of MIZU, USUD and from EV-network station JA01 were compared with those stemming from the PPP approach implemented in the software developed at NRCan [62].

For each station, the two solutions were aligned (i.e. their difference was set to zero) at a conventional initial epoch and compared within a time interval of 4 minutes (i.e. this interval was chosen according to the earthquake duration). At first, the retrieved positions were differenced at the corresponding epochs and the agreement between the solutions was evaluated by means of the RMSE of the differences. Then, the overall 4 minutes interval was split in 4 parts (each part 1 minute long) in order to inspect the agreement reliance with the earthquake du-

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1 This work was made possible thanks to the precious support of Dr. Henton Joe and Dr. Dragert Herb, NRCan, who produced the PPP solutions for the aforementioned stations

2 $\text{RMSE} = \sqrt{\text{bias}^2 + \text{st.dev.}^2}$

3 For the generic epoch $i$ the difference is given as: $\text{DIFF}_i = VADASE_i - \text{PPP}_i$
ration and to evaluate the impact of the drift affecting the variometric solutions. Finally, the two approaches were compared in the determination of the peak to peak displacement, which is an important parameter to estimate the earthquake magnitude ($M_w$) and, consequently, to eventually raise a tsunami warning.

The agreement between VADASE and PPP showed to be clearly dependent on the time interval length. The RMSE of the differences between the two solutions ranged from $0.01 \div 0.02$ m in East and North and $0.06$ m in Up, after one minute, to approximately $0.05$ m in East and North and (with much larger variability) $0.20$ m in Up after four minutes. Here, it is worth to underline that the dependency of the agreement on the earthquake duration mostly regards the vertical component. On the contrary, the agreement between the two approaches evaluated in terms of peak to peak displacements appeared independent from the earthquake duration. In details, the differences reached approximately $0.01$ m in East and North and $0.03$ m in Up components. Finally, the correlation coefficient between the two solutions was always higher than 99% for the planimetric components (but for the East component of USUD and the North component of JA01 which showed 97% and 96% correlations, respectively). The vertical component showed slightly lower (and with larger variability) correlations: 65%, 84% and 90% for stations MIZU, USUD and JA01, respectively.

VADASE is subject of a pending patent of “Sapienza” University of Rome ever since June 2010. In October 2010 VADASE was recognized as a simple and effective approach towards real-time coseismic displacement waveform estimation and it was awarded the DLR Special Topic Prize and the First Audience Award in the ESNC 2010. Then, a fruitful cooperation began with DLR with the aim of improving VADASE algorithm (e.g. extension to the future GALILEO constellation) and definitely demonstrating its real-time capabilities. In this regard, extensive tests were carried out using synthetic data produced by Spirent GNSS signal simulator. First, starting from the simplest possible condition to be simulated (i.e. signals traveling without any disturbances along the geometric path from the generic satellite to the receiver), each component perturbing the signal travel path was simulated separately in order to determine its influence over the waveforms estimation. The achieved results allowed the algorithm to be corrected, enriched and improved where needed. Then, the impact of some relativistic effects on the solutions was analyzed in details. In addition, an effective data snooping technique was implemented in order to strengthen the algorithm with respect to rapid, sharp changes in the atmospheric conditions (i.e. tropospheric and ionospheric fronts, ionospheric scintillation). Finally, VADASE was used to analyze, completely in real-time, data streams coming from one of the permanent GPS stations of DLR EV-network.
Outlook

The variometric approach presented in this work was originally designed to detect in real-time the displacements of a unique, stand-alone GPS receiver. As a matter of fact, all the results achieved so far and described within this work were obtained from the post processing of data archived in the form of RINEX observations files. Nonetheless, it is important to underline that the necessary information used by the variometric approach (i.e. observations and broadcast products) are immediately available in real-time to the receiver. Further, receiver velocities are solved with a rather insignificant computational burden (see previous considerations about processing times) that, in fact, is definitely compliant with modern receivers firmware capabilities. Hence, at present there are no constraining factors with respect to the implementation of the variometric algorithm in GNSS receivers firmware. At the moment of this writing, future plans have already been made in order to design a prototypal instrument that adopts the variometric algorithm for the real-time analysis of the recorded observations.

In the end, it is important to sketch out some issues, which can be addressed in the near future in order to better assess VADASE potentials and, possibly, to enlarge its applications:

- the joint processing of dual frequency observations coming from different constellations increases the number of available satellite and, as a consequence, improves the reliability of the described approach. Profiting from Spirent simulator capabilities, VADASE has been tested using observations from GPS and Galileo satellites. In this regard, some issues (e.g. small drifts in Galileo solutions) are not yet fully understood and require further investigations. Moreover, tests employing GLONASS observations are being elaborated at the moment of this writing and the extensions towards future systems (e.g. Compass) are foreseen

- exploiting the advantages provided by the high-rate acquisition, the variometric algorithm can be effective also using single frequency low-cost phase receivers. In this respect, some preliminary tests have already been analyzed but a further insight will be necessary

- VADASE applications can be widened considering the displacements monitoring of structures that are naturally subject to oscillations (e.g. skyscrapers, bridges, transmissions towers, ...)

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Appendix A

Sagnac effect

For the purposes of measuring and determining the orbits of GPS satellites, it is convenient to use an Earth Centered Inertial (ECI) coordinate system, in which the origin is at the center of mass of the Earth and whose axis are pointing in fixed directions with respect to the stars [45, p. 27]. The determination and the subsequent prediction of satellite orbits are carried out in an ECI reference frame. Nonetheless, for practical reasons (i.e. most GPS users are fixed to the Earth, or moving slowly over its surface), GPS has been design to send satellites broadcast orbital data in an ECEF reference frame, in which the Earth rotates with a fixed rotation rate defined by \( \dot{\Omega} = 7.2921151467 \times 10^{-5} \text{rad/s} \).

A generic user can apply the algorithm described in [44] and in table 2.3 to compute satellites’ position in such a frame. However, because of the Earth rotation during the signal transmission time, a relativistic error (namely the Sagnac effect) is introduced. This is due to the fact that a clock on the surface of the Earth will experience a finite rotation with respect to an ECI reference frame during the signal transmission time from the satellite to the receiver (figure A.1).

Among the various approaches to correct the Sagnac effect, here we apply the procedure described in [45, p. 308].

It is necessary to recall that an ECI frame can be obtained by freezing an ECEF frame at the time epoch where pseudorange measurements are made to the set of visible satellites. Since Sagnac effect does not arise in an ECI frame, each satellite position can be computed in terms of its ECEF coordinates \((X_{ECEF}, Y_{ECEF}, Z_{ECEF})\) using formulas in table 2.3 for the time epoch when the signal was emitted. Then, they can be transformed into the common ECI frame (defined above) using the rotation matrix described hereafter.
Figure A.1: Representation of the Sagnac effect

\[
\begin{bmatrix}
X_{ECI} \\
Y_{ECI} \\
Z_{ECI}
\end{bmatrix} = \begin{bmatrix}
\cos(\Omega \ast \tau) & \sin(\Omega \ast \tau) & 0 \\
-\sin(\Omega \ast \tau) & \cos(\Omega \ast \tau) & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
X_{ECEF} \\
Y_{ECEF} \\
Z_{ECEF}
\end{bmatrix}
\]

where \( \Omega \) is the Earth rotation rate and \( \tau \) is the signal transmission time. In addition, since the ECEF and the ECI frames were fixed at the epoch of signal reception, the receiver position will be the same in both frames.

More on different approaches to account for the Earth rotation correction, together with some numerical example, can be found in [6]. Here, it is shown how using a wrong model of satellites’ coordinates computation, which does not consider the Sagnac effect, would impact the variometric algorithm.

Let consider data simulated for scenario 1 (table 5.1) and let assume to compute satellites coordinates without applying the rotation matrix described above,
which corrects the Earth rotation effect. This produces a significant difference (around 100 meters) in 3D satellites’ position, as reported in table A.1. If these

<table>
<thead>
<tr>
<th>12000 epochs</th>
<th>Δ X [m]</th>
<th>Δ Y [m]</th>
<th>Δ Z [m]</th>
<th>Δ 3D [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>G01</td>
<td>-78.427</td>
<td>-115.064</td>
<td>0.000</td>
<td>139.249</td>
</tr>
<tr>
<td>G03</td>
<td>3.349</td>
<td>-124.958</td>
<td>0.000</td>
<td>125.003</td>
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<tr>
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<td>25.928</td>
<td>-126.716</td>
<td>0.000</td>
<td>129.341</td>
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<td>G14</td>
<td>82.398</td>
<td>-109.458</td>
<td>0.000</td>
<td>137.005</td>
</tr>
<tr>
<td>G16</td>
<td>-12.854</td>
<td>-114.503</td>
<td>0.000</td>
<td>115.222</td>
</tr>
<tr>
<td>G19</td>
<td>-32.777</td>
<td>-97.405</td>
<td>0.000</td>
<td>102.772</td>
</tr>
<tr>
<td>G20</td>
<td>-111.691</td>
<td>-89.217</td>
<td>0.000</td>
<td>142.950</td>
</tr>
<tr>
<td>G23</td>
<td>-77.042</td>
<td>-54.970</td>
<td>0.000</td>
<td>94.643</td>
</tr>
<tr>
<td>G31</td>
<td>83.255</td>
<td>-80.912</td>
<td>0.000</td>
<td>116.095</td>
</tr>
</tbody>
</table>

Table A.1: Differences in satellites coordinates not accounting for Sagnac effect.

wrong coordinates were used in the geometry terms of the variometric algorithm, the retrieved receiver displacement would be severely impacted, as it is displayed in figure A.2. On the left-hand y axis the red curves represents receiver displacements achieved with the correct model, whereas the blue curves result from wrong satellites’ positions stemming from neglecting Sagnac effect. The latter case leads to a 3D receiver displacement of 1.096 m in 10 minutes while the former returns a displacement of 0.003 m, mostly dependent on the receiver noise. On the right-hand y axis the number of observations used is reported in green.

Again, as happened in example described in section 5.1.3, it is interesting to note that receiver displacements exhibit a trend which is not dependent on the number of the observations, as it was the case when an uncertainty level was applied to the receiver initial position (figure 5.9).
Figure A.2: Receiver waveforms not accounting for Sagnac effect
Appendix B

Real-time VADASE application in the EV-network

The EV-network is a near real-time network of GNSS receiver based on modular, configurable and adaptable hardware and software components. EV-network is deployed and managed by DLR and it is considered as a Resarch and Development infrastructure to complement the functionality, performance, verification and operation of existing and future GNSS systems. The main functionality is to acquire and process GNSS signals as well as corresponding environmental information. The structure of EV-network implies the possibility to provide all the acquired data to internal and external users of the network by specific services.

With DLR support, and using the Application Programming Interface (API) specifically designed for the integration of user softwares in the EV-network, VADASE algorithm has been tuned to operate in real-time on the high-rate data stream broadcast via Transmission Control Protocol/Internet Protocol (TCP/IP) from the permanent GNSS stations of the network. A graphical interface displaying the real-time estimated receiver velocities was realized. Figure B.1 shows the real-time application\(^1\) of the variometric algorithm over the high-rate (20 Hz) data stream broadcast from Braunschweig, Germany station.

\(^1\)The results reported in this section are obtained processing GPS carrier phase observations
Figure B.1: Real-time application of VADASE algorithm over data stream (20 Hz) from Braunschweig permanent station, EV-network
## Acronyms

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>API</td>
<td>Application Programming Interface.</td>
</tr>
<tr>
<td>APL</td>
<td>Applied Physics Laboratory.</td>
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<tr>
<td>ASCII</td>
<td>American Standard Code for Information Interchange.</td>
</tr>
<tr>
<td>CMC</td>
<td>Code Minus Carrier.</td>
</tr>
<tr>
<td>CODE</td>
<td>Center for Orbit Determination in Europe.</td>
</tr>
<tr>
<td>CRTN</td>
<td>California Real Time Network.</td>
</tr>
<tr>
<td>DLR</td>
<td>German Aerospace Agency.</td>
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<tr>
<td>DoD</td>
<td>Department of Defence.</td>
</tr>
<tr>
<td>DOP</td>
<td>Dilution of Precision.</td>
</tr>
<tr>
<td>DOY</td>
<td>Day Of the Year.</td>
</tr>
<tr>
<td>DPRI</td>
<td>Disaster Prevention Research Institute.</td>
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<tr>
<td>DS</td>
<td>Data Snooping.</td>
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<tr>
<td>ECEF</td>
<td>Earth Centered Earth Fixed.</td>
</tr>
<tr>
<td>ECI</td>
<td>Earth Centered Inertial.</td>
</tr>
<tr>
<td>EEW</td>
<td>Earthquake Early Warning.</td>
</tr>
<tr>
<td>EGNOS</td>
<td>European Geostationary Navigation Overlay Service.</td>
</tr>
<tr>
<td>EOPs</td>
<td>Earth Orientation Parameters.</td>
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<td>ESNC</td>
<td>European Satellite Navigation Competition.</td>
</tr>
<tr>
<td>FOC</td>
<td>Full Operational Capability.</td>
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<tr>
<td>ftp</td>
<td>File Transfer Protocol.</td>
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<tr>
<td>GBAS</td>
<td>Ground-based Augmentation Systems.</td>
</tr>
<tr>
<td>Notation</td>
<td>Description</td>
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<tr>
<td>GDGPS</td>
<td>Global Differential GPS.</td>
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<tr>
<td>GEO</td>
<td>Group on Earth Observations.</td>
</tr>
<tr>
<td>GEONET</td>
<td>GPS Earth Observation Network.</td>
</tr>
<tr>
<td>GGTO</td>
<td>GPS to Galileo Time Offset.</td>
</tr>
<tr>
<td>GIPSY</td>
<td>GPS Inferred Positioning SYstem.</td>
</tr>
<tr>
<td>GLONASS</td>
<td>GLObal’naya NAVigatsionnay Sputnikovaya Sistema.</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System.</td>
</tr>
<tr>
<td>IAG</td>
<td>International Association of Geodesy.</td>
</tr>
<tr>
<td>IGS</td>
<td>International GNSS Service.</td>
</tr>
<tr>
<td>INGV</td>
<td>Istituto Nazionale di Geofisica e Vulcanologia.</td>
</tr>
<tr>
<td>ITRF</td>
<td>International Terrestrial Reference Frame.</td>
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<tr>
<td>JPL</td>
<td>Jet Propulsion Laboratory.</td>
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<td>JPO</td>
<td>Joint Program Office.</td>
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<tr>
<td>LLR</td>
<td>Lunar Laser Ranging.</td>
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<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration.</td>
</tr>
<tr>
<td>NAVSTAR</td>
<td>NAVigation Satellite Time and Ranging.</td>
</tr>
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<td>NRCan</td>
<td>Natural Resources Canada.</td>
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<td>NRL</td>
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<td>PBO</td>
<td>Plate Boundary Observatory.</td>
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<td>PDOP</td>
<td>Position Dilution of Precision.</td>
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<tr>
<td>PPP</td>
<td>Precise Point Positioning.</td>
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<tr>
<td>PRN</td>
<td>Pseudorandom Noise.</td>
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<tr>
<td>RF</td>
<td>Radio Frequency.</td>
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<tr>
<td>RINEX</td>
<td>Receiver INdependent EXchange format.</td>
</tr>
<tr>
<td>RING</td>
<td>Rete Integrata Nazionale GPS.</td>
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<tr>
<td>RMSE</td>
<td>Root Mean Square Error.</td>
</tr>
<tr>
<td>RTK</td>
<td>Real-Time Kinematic.</td>
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<tr>
<td>RTPP</td>
<td>Real-Time Pilot Project.</td>
</tr>
<tr>
<td>S/A</td>
<td>Selective Availability.</td>
</tr>
<tr>
<td>SAMSO</td>
<td>Space and Missile System Organization.</td>
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<td>Notation</td>
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<td>----------</td>
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<tr>
<td>SBAS</td>
<td>Space-based Augmentation Systems.</td>
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<tr>
<td>SLR</td>
<td>Satellite Laser Ranging.</td>
</tr>
<tr>
<td>TEC</td>
<td>Total Electron Content.</td>
</tr>
<tr>
<td>TECU</td>
<td>TEC Units.</td>
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<tr>
<td>TID</td>
<td>Traveling Ionospheric Disturbances.</td>
</tr>
<tr>
<td>UNAVCO</td>
<td>University NAVSTAR Consortium.</td>
</tr>
<tr>
<td>US</td>
<td>United States.</td>
</tr>
<tr>
<td>UTC</td>
<td>Coordinated Universal Time.</td>
</tr>
<tr>
<td>VADASE</td>
<td>Variometric Approach for Displacement Analysis Stand-alone Engine.</td>
</tr>
<tr>
<td>VLBI</td>
<td>Very-Long Baseline Interferometry.</td>
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<tr>
<td>WAAS</td>
<td>Wide-area Augmentation Systems.</td>
</tr>
<tr>
<td>ZTD</td>
<td>Zenith Tropospheric Delay.</td>
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Acknowledgments

This work is the result of three years of research. Expressing my gratitude to all the people that helped me, in all various and different meanings that the term help encompasses, would require a couple of days (and a few tens of pages) more. However, for the sake of shortness, I will limit myself.

First of all, I would like to express a special thank to my supervisor, Professor Mattia Crespi, who led me throughout this experience. He has always been keen on guiding and supporting my activities, showing true curiosity with respect to my research and being an outstanding and continuous source of ideas and suggestions. A part from the topic described here, I had the chance to work with him in different fields and I truly believe this enriched and widened my knowledge and experience. Overall, he fundamentally contributed to enhance these three years of research, and not only from a scientific standpoint.

Then, I am totally aware that this work substantially benefited from the precious help of Dr. Augusto Mazzoni and from his wide knowledge of many topics related to GNSS. Working and sharing this (and other) experience(s) with him was a pleasure and a true satisfaction.

I would like to thoroughly thank Dr. Thomas Dautermann, my co-supervisor. He warmly welcomed me at the German Aerospace Agency (DLR), being a fundamental source of informations and advices within my whole stay. We designed and realized a field experiment that required a lot of work, being instructive, useful, challenging and, as a matter of fact, funny and enjoyable.

I am grateful to all members of the Institute of Communications and Navigation that welcomed me and made my stay comfortable, productive and fruitful.
I shared the last three years of my working and studying experience with other researchers and Ph.D. candidates of the Geodesy and Geomatics Area at “Sapienza” University of Rome. The nice, productive, calm and cooperative atmosphere that I experienced is due to each of them. Thank you.

Finally, being done with the (mostly) scientific thanks, I would like to acknowledge all the people the supported, fostered and boosted me, day by day, towards this goal: Family, Friends, Love (the order being merely alphabetical). All of you, to a different extent, took a big role in this adventure. I owe you a lot.
Bibliography


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